Some of these exercises instruct you to quote a p-value in your conclusion. Regard p-values as optional; you should, however, always draw a conclusion.

1. Exercise 10.28 in the text.

   Solution:

   Based on the normality and common variance assumptions, we should do a pooled t-test.

$$s_p^2 = \frac{24 * 1.5^2 + 24 * 1.25^2}{25 + 25 - 2} = 1.90625$$

   so that $s_p = 1.38067$. Then, the value of the pivot under H_0 (also called the "test statistic") is

$$\frac{\bar{x}_n - \bar{y}_n - (\mu_x - \mu_y)}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{20 - 12 - 0}{1.38067\sqrt{1/25 + 1/25}} = 20.4859.$$

   Since the alternative hypothesis is $H_a : \mu_x > \mu_y$, we compare the above value to the percentile $t_{0.95,48} \approx 1.678$ and we clearly reject $H_0 : \mu_x = \mu_y$.

2. Exercise 10.30 in the text.

   Solution:

   In this case, since we know the true population standard deviations (or variances) we use a z-test, that is, a test based on the percentiles of the normal distribution:

$$\frac{\bar{x}_n - \bar{y}_n - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} = \frac{81 - 76 - (0)}{\sqrt{\frac{5.2^2}{25} + \frac{3.4^2}{36}}} = 4.221686.$$

Based on the alternative hypothesis $H_a : \mu_x \neq \mu_y$ we compare to both $z_{0.025} = -1.96$ and $z_{0.975} = 1.96$ for $\alpha = 0.05$. Clearly, we reject H_0. Since $1 - P(Z < 4.22) \approx 0$ the p-value is almost zero.

3. Exercise 10.32 in the text.

Solution:

Since the number of samples (200) in each group is so large, we can use an approximate test based on normal percentiles.

$$\frac{\bar{x}_n - \bar{y}_n - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} = \frac{70750 - 65200 - (2000)}{\sqrt{\frac{6000^2}{200} + \frac{5000^2}{200}}} = 6.428038.$$

The z critical value for comparison is $z_{0.99} = 2.325$. Clearly we will reject H_0.

4. Exercise 10.34 in the text.

Solution:

Based on the assumptions of normality and equal variances, we should use the pooled t-test. Our hypotheses are $H_0 : \mu_x - \mu_y = 8$ versus $H_a : \mu_x - \mu_y < 8$.

$$s_p^2 = \frac{10 * 4.7^2 + 16 * 6.1^2}{11 + 17 - 2} = 31.39$$

so that $s_p = 5.60309$. Then, the value of the pivot under H_0 (also called the "test statistic") is

$$\frac{\bar{x}_n - \bar{y}_n - (\mu_x - \mu_y)}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{85 - 79 - 8}{5.60309\sqrt{1/11 + 1/17}} = -.922.$$

Since the alternative hypothesis is $H_a : \mu_x - \mu_y < 8$, we compare the above value to the percentile $t_{0.05,26} = -1.706$ and we do not reject. The p-value is $P(t_{26} < -.922) = .1825$.

5. Exercise 10.40 in the text.

Solution:

Based on the assumptions of normal data with unequal and unknown variances, we do the approximate t-test with calculated degrees of freedom. The hypotheses are $H_0 : \mu_x = \mu_y$ vs. $H_a : \mu_x \neq \mu_y$. Degrees of freedom is calculated with the formula:

$$v = \frac{(\frac{s_x^2}{n} + \frac{s_y^2}{m})^2}{\frac{(s_x^2/n)^2}{n-1} + \frac{(s_y^2/m)^2}{m-1}} = \frac{(\frac{0.391478^2}{8} + \frac{0.214414^2}{24})^2}{\frac{(0.391478^2/8)^2}{8-1} + \frac{(0.214414^2/24)^2}{24-1}} = 8.$$

The test statistic is $\frac{\bar{x}_n - \bar{y}_n - (\mu_x - \mu_y)}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = \frac{0.97625 - 0.91583}{\sqrt{0.391478^2/8 + 0.214414^2/24}} = -.42$ and we fail to reject H_0.

6. Exercise 10.44 in the text.

Solution:

This is an example of paired data, so we need to do a paried t-test.

$$\frac{\bar{d}}{s/\sqrt{n}} = \frac{198.625}{210.1652/\sqrt{8}} = 2.673118.$$

To test $H_0 : \mu_x - \mu_y = 0$ vs. $H_a : \mu_x - \mu_y \neq 0$ we compare to $t_{7,0.025} = 2.365$ at the $\alpha = 0.05$ level. Reject H_0.