Math 516: Exam 1

Problem 1. (25 points) Let $G$ be a group and let $H$ and $K$ be normal subgroups of $G$ with $H \leq K$. Prove that $K/H \unlhd G/H$ and $(G/H)/(K/H) \cong G/K$.

Problem 2. (20 points) Let $G$ be a group. Let $H$ be a subgroup of $G$ such that $C_G(H) = G$ and $G/H$ is cyclic. Show that $G$ is abelian.

Problem 3. (40 points) Let $G$ be a finite group and $p$ be the smallest prime dividing $|G|$. Let $H$ be a subgroup of $G$.
   a) Show that if $|G : H| = p$ then $H$ is a normal subgroup of $G$.
   b) Show that if $|H| = p$ and $H$ is normal in $G$ then $H$ is contained in the center of $G$.

Problem 4. (15 points) Give the number of nonisomorphic abelian groups of order 144.