Math 215 Sample problems for Test 2.

1. Determine for which values a, b, c the finction

$$f(x) = ax^2 + bx + c : \mathbb{R} \to \mathbb{R}$$

   is injective, surjective, and bijective.

2. Prove that if there is a surjection $f : \mathbb{N}_n \to X$, then X is finite and $|X| \leq n$.

3. Let X be a finite set with $|X| = n$. Let $A \subset X$ be a fixed subset of X of cardinality m. Count the number of subests of X containing A. Count the number of subsets having empty intersection with A. Count the number of subsets whose intersection with A has cardinality $k \leq m$.

4. Construct a bijection between $\mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$.

5. Recover the rational number x from its decimal representation if

$$x = 12.30\overline{104}$$

$$x = 0.964\overline{21}.$$

6. Show that a is even if and only if a^3 is even.

7. Find the gcd of 45 and 18.