Pr. 1. Injective. As long as $a \neq 0$, f is not injective. So, $a = 0$ is the first requirement. Then f is a linear function. it is injective iff $b \neq 0$. So, $a = 0$ and $b \neq 0$.

Surjective. The same conditions.

Bijective. The same conditions.

Pr. 2. Constructive approach. For every $x \in X$, let $A_x = \{m \in \mathbb{N}_n : f(m) = x\}$. Since f is surjective A_x is nonempty for every x. Let us pick one element $m_x \in A_x$ for every $x \in X$. And let us define the map $h : X \in \mathbb{N}_n$ by the rule $h(x) = m_x$. Then h is injective. If, $|X| > n$, then by the pigenhole principle, h could not be injective. Therefore, $|X| \leq n$.

Pr 3. The number of subsets containing A is the same as the number of subsets of $X - A$, which is 2^{n-m}. Second question has the same answer. Third, the number of such subsets is equal to the product of the number of subsets of A of cardinality k, which is $m!/k!(m-k)!$, and the total number of all subsets of $X - A$, which is 2^{n-m}. So the answer is

$$\frac{2^{n-m} m!}{k!(m-k)!}.$$

Pr4. One approach is to establish a bijection is to arrange $\mathbb{Z} \times \mathbb{Z}$ into an infinite array stratched to infinity left right up and down, and start counting with $(0,0)$ at the center, then go to $(0,1)$ and count the entries clockwise. Then move to the next circle starting with $(0,2)$ and count clockwise.

Another approach is to split $\mathbb{Z} \times \mathbb{Z}$ into four quadrants $\mathbb{N} \times \mathbb{N}$, $\mathbb{N}^- \times \mathbb{N}$, etc., and count each using the same scheme as ws presented in class, but for each quarant using naturals which are divisible by 4 with remainders $0, 1, 2, 3$, respectively.

There are plenty of solutions to the this problem.

Pr5. Just do what we did in class.

Pr6. If $a = 2q$, then $a^3 = 2 \cdot 4q^2$. If a^3 is even, then a cannot be odd as well. Indeed, if it is, then $a = 2q+1$, then $a^3 = 8q^3 + 12q^2 + 6q + 1 = 2(4q^3 + 6q^2 + 3q) + 1$, contradiction.

Pr7. 9.