Math215: Sample problems for Exam 1

1. Write the negation of the following statement

   *Some people exercise for at least 30 minutes every day.*

2. a) Use order axioms to show that for real numbers $x$ and $y$,

   $$(x > 0 \land y > 1) \Rightarrow \left( \frac{x}{y} < x \right)$$

   b) Prove that there does not exist a smallest positive real number.

3. Prove by induction that the sum of the squares of integers satisfies

   $$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{(2n + 1)(n + 1)n}{6}$$

   for all $n \geq 1$.

4. Let $A$, $B$, and $C$ be sets. Prove that $(A \cup B) - C \subseteq (A - B) \cup C$. Draw Venn diagrams to illustrate the proof. Give an example of sets $A$, $B$ and $C$ such that equality (rather than simply $\subseteq$) holds. Give an example where equality does not hold.

5. Prove or disprove the following statements

   a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \ x^2 < y$
   
   b) $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, \ x^2 < y$
   
   c) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, \ x^2 < y$
   
   d) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, \ x^2 > y$