Math215: Sample problems for Exam 1

1. Write the negation of the following statement

   $$\textit{Some people exercise for at least } 30 \textit{ minutes every day.}$$

2. a) Use order axioms to show that for real numbers x and y,

   $$(x > 0 \wedge y > 1) \Rightarrow (\frac{x}{y} < x)$$

   b) Prove that there does not exist a smallest positive real number.

3. Prove by induction that the sum of the squares of integers satisfies

   $$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{(2n+1)(n+1)n}{6}$$

   for all $n \geq 1$.

4. Let A, B, and C be sets. Prove that $(A \cup B) - C \subseteq (A - B) \cup C$. Draw Venn diagrams to illustrate the proof. Give an example of sets A, B and C such that equality (rather than simply $\subseteq$) holds. Give an example where equality does not hold.

5. Prove or disprove the following statements

   a) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x^2 < y$

   b) $\forall y \in \mathbb{R},\ \exists x \in \mathbb{R},\ x^2 < y$

   c) $\exists y \in \mathbb{R},\ \forall x \in \mathbb{R},\ x^2 < y$

   d) $\exists y \in \mathbb{R},\ \forall x \in \mathbb{R},\ x^2 > y$