

MATH 220 EXAM II

C. Tier

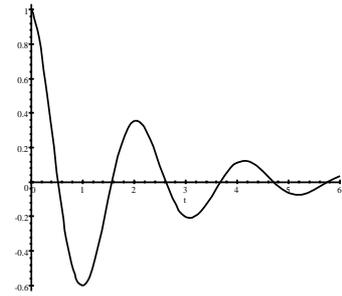
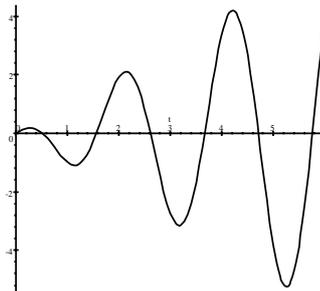
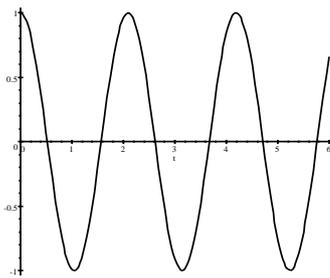
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Answer all questions (25pts/question) - **show all work in your exam booklet.**

1. (a) Find $\mathcal{L}\{e^{tt^3}\}$
 - (b) Use the convolution theorem to find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$
 - (c) Find $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$
2. Consider the undamped oscillator with displacement $x(t)$ governed by:

$$x'' + 4x = 2 \cos \gamma t$$

- (a) Find an explicit formula for a particular solution when $\gamma \neq 2$.
- (b) Choose and re-sketch in your exam booklet the graph which best represents the behavior when $\gamma = 2$. What is the significance of $\gamma = 2$?



3. Find $y(t)$:

$$y' + 4y = 3e^{-4t} - 2u(t-2), \quad y(0) = 5$$

4. Find $x(t)$:

$$\begin{aligned} x' &= -2x + y + \delta(t), & x(0) &= 0, \\ y' &= 3x - 4y, & y(0) &= 1 \end{aligned}$$

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
$e^{at}f(t)$	$F(s-a)$
$f'(t)$	$sF(s) - f(0)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$
$\int_0^t f(v)dv$	$F(s)/s$
$f * g$	$F(s)G(s)$
$f(t-a)u(t-a), a \geq 0$	$e^{-as}F(s)$
$g(t)u(t-a), a \geq 0$	$e^{-as}\mathcal{L}\{g(t+a)\}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
$\sinh bt$	$\frac{b}{s^2 - b^2}$
$\cosh bt$	$\frac{s}{s^2 - b^2}$