Math 220 **Final Exam** **December 12, 2002**

- Show **all** of your work. An unjustified answer is not correct!
- Put all work and solutions in the paper provided, with a BOX around your final answers.
- Please indicate if you use a TI-89/92. calculator. You are required to show intermediate steps.
- **Write on the first page of the exam booklet or solutions** your
  1. **NAME**
  2. **SOCIAL SECURITY NUMBER**
  3. **LECTURER NAME (HURDER or WOOD)**
  4. **DISCUSSION SECTION HOUR/DAY**

Keep your eyes on your own work and keep your work covered.
A table of Laplace transforms and useful formulae is provided for your use.

There are 10 problems - 5 on this side, and 5 on the other side.

Problem 1: (20 pts) Find the general solution of the ODE

$$y'' + 2y' + 5y = 0$$

Problem 2: (20 pts) Find the general solution of the ODE

$$y'' + 4y = \cos 3t$$

Problem 3: (20 pts) Find the solution of the ODE

$$\frac{dy}{dx} = \sqrt{y},\ y(0) = 1$$

Problem 4: (20 pts) Find the solution of the ODE

$$\frac{dy}{dx} + \frac{1}{x}y = 1,\ y(1) = 1$$

Problem 5: (20 pts) Find the power series through the x^5 term for the solution $y(x)$ of the equation

$$y' + xy + x = 0,\ \ y(0) = 1$$

Problem 6: Given the initial value problem (where $u(t)$ is the Heaviside step function):

$$y' + 2y = u(t-3), \;\; y(0) = 1$$

a) (10 pts) Find the Laplace transform $Y(s)$ of the solution $y(t)$

b) (10 pts) Use the tables of Laplace transforms to find the solution $y(t)$ using your answer to a)

Problem 7: Let $f(x) = 1$ for $0 < x < \pi$.

a) (5 pts) Sketch the graph of the **ODD** function which extends f to $-\pi < x < \pi$

c) (15 pts) Find the Fourier sine series for f on $0 < x < \pi$

Problem 8: (20 pts) Find the solution of the heat equation

$$\frac{\partial u(x,t)}{\partial t} = 2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < \pi, \;\; t > 0;$$
$$u(0,t) = 0, u(\pi,t) = 0, t > 0$$
$$u(x,0) = \sin x + \frac{1}{3}\sin(3x), 0 < x < \pi$$

Problem 9: (20 pts) Find the solutions $x(t)$ and $y(t)$ of the system

$$\begin{aligned} x' &= 4x - 3y \\ y' &= x \end{aligned}$$

with initial conditions $x(0) = -2$ and $y(0) = 5$.

Problem 10: (20 pts) Given the ODE:

$$\frac{dy}{dt} = -ty + y^2, \; y(0) = 1$$

Use Euler's numerical algorithm with step size $h = 1/10$ to compute $y(0.1)$ and $y(0.2)$.