

### 3 Average Values

Let us start by recalling how averages work with a finite, discrete set of data. For example, say we have the following test scores: 60, 85, 72, 87, 75, 92; if we want to find their average, we can simply add them all up and divide by the total number of scores, in this case 6. That is, if we call the average value  $A$ , we have  $A = \frac{1}{6}(60 + 85 + 72 + 87 + 75 + 92) = \frac{1}{6}(474) = 79$ .

The question we may now want to answer is how we find the average value of some continuous function. For example, say we have a function  $c(t)$  representing the cost of some item over time and we want to know the average price over some time period. We cannot simply add up all of the prices and divide by the amount of time, since the price varies continuously over time. However, we can use integrals to find the total area underneath  $c(t)$ , which we can interpret as being a continuous version of adding up all of the prices. We can then divide this total by the length of time to get our average value.

This is a very common theme in calculus – using limits to turn something discrete into something continuous. We will see this come up again later in the semester, so it is useful to keep in mind – we can think of integrals as being like “continuous sums.”

Let’s compute an example. Suppose we want to find the average value of  $f(x) = 3x^2 - 2$  on the interval  $[1, 3]$ . We can start by computing the area underneath  $f(x)$  between 1 and 3, by computing the definite integral

$$\begin{aligned}\int_1^3 (3x^2 - 2)dx &= (x^3 - 2x)\Big|_1^3 \\ &= (3^3 - 2(3)) - (1^3 - 2(1)) \\ &= 21 - (-1) \\ &= 22\end{aligned}$$

Then, to find the average value we divide this by the total length of the interval, which is  $3 - 1 = 2$ , which gives us an average value of  $\frac{22}{2} = 11$ .

More generally, to find the average value of  $f(x)$  on an interval  $[a, b]$ , we calculate the total area underneath  $f$  between  $a$  and  $b$  and then divide that by the length of the interval, which is  $b - a$ . That is, the average value can be computed using the formula

$$\frac{1}{b-a} \int_a^b f(x)dx$$