3 Substitution

Problem 3.1.

***a*)** Compute $\int 2x^3 dx$ using the power rule.

***b*)** Notice that $2x^3 = 2x \cdot x^2$. Let $u = x^2$ and again compute $\int 2x^3 dx$, but this time using substitution.

***c*)** Now, using part a, compute the definite integral $\int_0^2 2x^3 dx$.

***d*)** To compute the same definite integral using the substitution from part b, what should the bounds be? (Hint: if $u = x^2$, when $x = 0$ what does u equal? What about when $x = 2$?)

***e*)** Check that your answers in parts c and d are the same. If not, double check all your work.

Problem 3.2. *Use substitution to find the following indefinite integrals:*

a) $\int x^2 \sin(x^3) dx$

b) $\int \cos(x) \sin(x) dx$

c) $\int \frac{2x}{\sqrt{x^2+9}} dx$

d) $\int \frac{2x-1}{x^2-x} dx$

e) $\int \cos(3x) e^{\sin(3x)} dx$

f) $\int (x^7+2)(x^8+16x-3)^5 dx$

(!) ***g)*** $\int (x^2\cos(x^3+3x) + \cos(x^3+3x)) dx$

(!) ***h)*** $\int \frac{3x^2 e^{\sqrt{x^3-5}}}{\sqrt{x^3-5}} dx$

Problem 3.3. *Use substitution to find the following definite integrals:*

a) $\int_0^1 (x^2+1)(x^3+3x)^4 dx$

b) $\int_0^{\pi} 2\sin(2x)e^{\cos(2x)} dx$

c) $\int_{-1}^{2} x^2 e^{x^3-1} dx$

d) $\int_1^2 \frac{x^2+x}{\sqrt{x+1}} dx$

e) $\int_0^{\frac{\pi}{4}} \ln(\cos(x))\tan(x) dx$

f) $\int_0^{\frac{3\pi}{2}} \cos(x)\sin(\sin(x)) dx$

(!) ***g)*** $\int_{-\frac{\sqrt{3}-1}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{2x^2-1}{(2x^3-3x)^2}\arctan(2x^3-3x) dx$

(!) ***h)*** $\int_a^b f(g(x))g'(x) dx,\ F = \int f dx$