

# Written Homework # 8

Due at the beginning of class 08/05/2009

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1. Let  $n$  be a positive integer.

*Fact:* Suppose  $n$  is a perfect square. Then  $n = 5m$ , or  $5m + 1$ , or  $5m + 4$  for some non-negative integer  $m$ .

Using only the fact and Proposition 15.2.3 determine whether or not

- (a) 143 is a perfect square;
- (b) 100000012 is a perfect square.

2. Use the Division Algorithm to prove the following: If  $n$  is an integer then 5 divides  $n^2$  implies  $5^2$  divides  $n^2$ .

3. For integers  $a$  and  $b$  in each case below find the unique integers  $q, r$  which satisfy  $a = qb + r$  and  $0 \leq r < b$ :

- (a)  $a = 291$  and  $b = 28$ ;
- (b)  $a = -2933$  and  $b = 19$ .

4. Use the Euclidean algorithm to find the greatest common divisor of:

- (a) 231 and 95;
- (b) 840 and 220.