

1. (25 points total)

(a) $143 = 3 \cdot 47 + 2$; thus 143 is not a perfect square Proposition 15.2.3. (12)

(b) $100000012 = 3 \cdot 3333337 + 1$; thus Proposition 15.2.3 is inconclusive. (6) On the other hand, $100000012 = 5 \cdot 20000002 + 2$; thus 100000012 is not a perfect square by *Fact*. (7)

Comment: Testing using *Fact* leads to the same conclusion as $110693 = 5 \cdot 22138 + 3$.

2. (25 points total) Suppose that $n \in \mathbf{Z}$ and 5 divides n^2 . Write $n = 5m + r$, where $m, r \in \mathbf{Z}$ and $0 \leq r < 5$. Then $n^2 = (5m + r)^2 = 5^2m^2 + 2 \cdot 5mr + r^2 = 5(5m^2 + 2mr) + r^2$. Since 5 divides n^2 necessarily 5 divides r^2 . (15 points) Using the table

| | | | | | |
|-------|---|---|---|---|----|
| r | 0 | 1 | 2 | 3 | 4 |
| r^2 | 0 | 1 | 4 | 9 | 16 |

we see that 5 divides r^2 only when $r = 0$. Therefore $r = 0$ and $n = 5m$. (10 points)

3. (25 points total) (a) $291 = 28 \cdot 10 + 11$; $q = 10$ and $r = 11$ (10) (b) $-2933 = 19 \cdot (-155) + 12$; $q = -155$ and $r = 12$. (15)

4. (25 points total)

(a)

$$\begin{aligned}
 231 &= 95 \cdot 2 + 41 \\
 95 &= 41 \cdot 2 + 13 \\
 41 &= 13 \cdot 3 + 2 \\
 13 &= 2 \cdot 6 + 1 \\
 2 &= 1 \cdot 2 + 0.
 \end{aligned}$$

Therefore the greatest common divisor of 231 and 95 is 1. (15 points)

(b)

$$\begin{aligned}
 840 &= 220 \cdot 3 + 180 \\
 220 &= 180 \cdot 1 + 40 \\
 180 &= 40 \cdot 4 + 20 \\
 40 &= 20 \cdot 2 + 0.
 \end{aligned}$$

Therefore the greatest common divisor of 840 and 220 is 20. (10 points)