

Name (print) \_\_\_\_\_ Signature \_\_\_\_\_

UIC ID \_\_\_\_\_

(1) *Write* your name and your instructor's name on your examination booklet. (2) *Write* your answers in the exam booklet provided. (3) *Return* your copy of the examination with the examination booklet(s). (4) *Show* your work. **An unjustified answer receives no credit.** (4) There are **10** questions on this examination. Check to see that this copy is complete. (5) If you use a calculator it must be *your own*. (6) *You are expected to abide by the University's rules concerning academic honesty.*

(7) *Circle* your instructor's name and lecture hour:

Dias (8AM) Kobotis (9AM) Chakravorty (10AM) Radford (11AM) Masley (12PM)

Kashcheyeva (1PM) Tartakoff (1PM) Chakravorty (2PM) Ruan (3PM)

(8) *Circle* your discussion hour: 8AM 9AM 10AM 11AM 12PM 1PM 2PM 3PM



Do not write in this area.

1	2	3	4	5	6	7	8	9	10
<b>30</b>	<b>15</b>	<b>20</b>	<b>20</b>	<b>20</b>	<b>25</b>	<b>15</b>	<b>15</b>	<b>20</b>	<b>20</b>

SCORE \_\_\_\_\_ /200



1. (30) Find the derivatives of the following five functions; *do NOT simplify answers.*

(a)  $3^x + x^3 + \frac{x}{3} + \frac{3}{x}$       (b)  $\frac{x^4 - \cos x}{2x^6 + \tan x}$       (c)  $\sqrt{(\ln x)^2 + 5}$

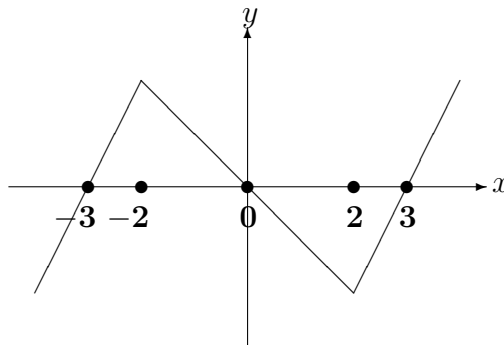
(d)  $(x^2 + 1)^{231}(1 + \sin x)$       (e)  $e^{3x} + \sinh 4x + \arctan x + \frac{1}{\sqrt{x}}$

2. (15) Let  $f(x) = 2x^2 - x - 1$ . Use the definition of the derivative as the limit of a difference quotient to compute  $f'(x)$ .

3. (20) The height of a ball thrown straight up into the atmosphere on a strange and exotic planet at time  $t$  seconds is  $s(t) = -6t^2 + 3t + 30$  meters. Use Calculus to answer the following questions.

- (a) Find the velocity and the acceleration of the ball at time  $t$ . Give the units for each.
- (b) When does the ball reach its maximum height?
- (c) What is its maximum height?
- (d) When does the ball hit the ground?

4. (20) The graph below is that of the derivative  $f'(x)$ . The domain of  $f(x)$  is  $(-4, 4)$ .



- (a) Find the values of  $x$  where a local minimum of  $f(x)$  occurs.
  - (b) Find the intervals on which the graph of  $y = f(x)$  is concave down.
  - (c) Find the  $x$ -coordinates of inflection points on the graph of  $y = f(x)$ .
5. (20) The function  $y = f(x)$  is defined implicitly by the equation  $y^3x^2 + xy - x^2 = 2$ .
- (a) Find an equation of the line tangent to the graph of  $y = f(x)$ , that is line tangent to the curve defined by the equation, at the point  $(2, 1)$ .
  - (b) Use tangent line approximation to estimate  $f(2.1)$ .
6. (25) Let  $f(x) = x^4 + x^3$ . Part (e) of this problem is on the next page.
- (a) Find the *critical points* of  $y = f(x)$ .
  - (b) Find the intervals on which  $f(x)$  is *increasing*, on which  $f(x)$  is *decreasing*.
  - (c) Find the intervals on which the graph of  $y = f(x)$  is *concave down*, on which the graph is *concave up*.
  - (d) Find the *inflection points* on the graph of  $y = f(x)$ .

- (e) Using the information derived from parts (a)–(d), sketch the graph of  $y = f(x)$ . Plot the points corresponding to local maxima and minima, inflection points, and points where the graph crosses the  $x$ -axis. *Coordinates of points must be expressed as integers or fractions, like  $-31$  or  $7/11$ .*

7. (15) Find:

(a)  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1 - 3x}{x^2}$  by L'Hopital's rule;

- (b) the unique real number  $c$  such that the function given by  $f(x) = \begin{cases} x^2 + 4x & : x \leq 3 \\ 2cx + 1 & : 3 < x \end{cases}$  is continuous at  $x = 3$ .

8. (15) Let  $f(x) = x^2 + x$ .

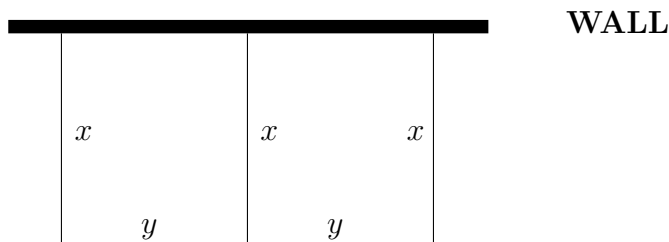
(a) Find the right hand sum (right Riemann sum) for  $\int_0^2 (x^2 + x) dx$  where  $n = 4$ .

- (b) Is the right hand sum of part (a) an underestimate or is it an overestimate of  $\int_0^2 (x^2 + x) dx$ ? Explain your answer.

9. (20) Use *Calculus* to find:

- (a) the average value of the function  $g(x) = x^3$  on the interval  $[-1, 3]$ ;
- (b) the area of the region bounded by the lines  $x = 0$ ,  $x = \ln 2$ , the  $x$ -axis, and the graph of  $y = f(x) = e^x + 2$ .

10. (20) A farmer wishes to construct two rectangular fenced pens of the same dimensions against a long wall as depicted in the diagram:



The area of *each* pen is to be 5,000 square feet. Fencing for the three sides perpendicular to the wall costs \$25 per foot and fencing for the side parallel to the wall costs \$12 per foot.

- (a) Express the total cost of the fencing  $C(x)$  as a function of  $x$  and find its domain.
- (b) Find the  $x$  such that the cost  $C(x)$  is minimal. *Your answer must be justified by Calculus.*
- (c) Find the minimal cost of the fencing.