Name (print) ___________________________ Discussion hour (T Th ______) 

1. (10 pts.)

a) Compute the derivatives of the functions :

- \( f(x) = 6 \sqrt[3]{x} + \frac{4}{\sqrt[3]{x}} \)

**Solution** : We can write the function as \( f(x) = 6 x^{\frac{1}{3}} + 4 x^{-\frac{1}{3}} \) and so, by using the power rule, we get:

\[
    f'(x) = 6 \left( x^{\frac{1}{3}} \right)' + 4 \left( x^{-\frac{1}{3}} \right)' = 6 \left( \frac{1}{3} \right) x^{-\frac{2}{3}} + 4 \left( -\frac{1}{4} \right) x^{-\frac{4}{3}} = 2 x^{-\frac{2}{3}} - x^{-\frac{4}{3}}.
\]

So answer is \( f'(x) = 2 x^{-\frac{2}{3}} - x^{-\frac{4}{3}} \) or \( f'(x) = \frac{2}{x^{2/3}} - \frac{1}{x^{4/3}} \). (3 pts.)

(Equivalent forms of final answer are acceptable and get full credit.)

- \( g(x) = \frac{\sqrt{x}(1 + 2x)}{x^2} \)

**Solution** : We can write the function in the following way :

\[
    g(x) = \frac{\sqrt{x}(1 + 2x)}{x^2} = \frac{x^{\frac{1}{2}}(1 + 2x)}{x^2} = x^{\frac{1}{2}-2} + 2x^{\frac{3}{2}-2} = x^{-\frac{3}{2}} + 2x^{-\frac{1}{2}}.
\]

So, by using the power rule we get \( g'(x) = -\frac{3}{2} x^{-\frac{3}{2}} - x^{-\frac{3}{2}} \). (3 pts.)

(Again, equivalent forms of final answer are acceptable and get full credit.)

b) Find the equation of the line tangent to the graph of \( f \) at \( x = 1 \).

**Solution** : We are interested in the point \( (1, f(1)) \) i.e. \( (1, 10) \). The slope at that point is \( m = f'(1) = 1 \) (using the formula of \( f' \) that we computed above). So the equation of the tangent line at that point is : \( y - 10 = x - 1 \Rightarrow y = x + 9 \). (4 pts.)

2. (10 pts.)

a) Compute the derivative of the function : \( h(x) = 2\sqrt{x} - \left( \frac{1}{3} \right)^x \).

**Solution** : \( h'(x) = 2 \left( x^{\frac{1}{2}} \right)' - \left[ \left( \frac{1}{3} \right)^x \right]' \Rightarrow h'(x) = x^{-\frac{1}{2}} - \left( \ln \frac{1}{3} \right) \left( \frac{1}{3} \right)^x \). (5 pts.)
b) Suppose that \( P(t) = 50 \times (1.2)^t \), expresses the growth of price \( P \) (in dollars) as a function of time \( t \) (in years). Find the rate, in dollars per year, at which the price \( P \) is increasing.

**Solution**: We actually want to find the derivative function \( P' \). A direct differentiation gives:

\[
P'(t) = 50 \left(\ln 1.2 \times 1.2\right)^t \Rightarrow P'(t) = 50(\ln 1.2)(1.2)^t \approx 9.12(1.2)^t.
\]

(5 pts.)
1. (10 pts.)
   a) Compute the derivatives of the functions :
      
      \[ f(x) = 8\sqrt[4]{x} + \frac{3}{\sqrt[3]{x}} \]
      
      **Solution** : We can write the function as
      \[ f(x) = 8x^{\frac{1}{4}} + 3x^{-\frac{1}{3}} \]
      and so, by using the power rule, we get:
      \[ f'(x) = 8\left(\frac{1}{4}\right)x^{-\frac{3}{4}} + 3\left(-\frac{1}{3}\right)x^{-\frac{4}{3}} = 2x^{-\frac{3}{4}} - x^{-\frac{4}{3}}. \]
      
      So answer is \[ f'(x) = 2x^{-\frac{3}{4}} - x^{-\frac{4}{3}} \] or \[ f'(x) = \frac{2}{x^{3/4}} - \frac{1}{x^{4/3}}. \] (3 pts.)
      
      (Equivalent forms of final answer are acceptable and get full credit.)

      \[ g(x) = \frac{\sqrt[3]{x}(1 + 2x^{\frac{2}{3}})}{x^2} \]
      
      **Solution** : We can write the function in the following way :
      \[ g(x) = \frac{x^{\frac{1}{3}}(1 + 2x^{\frac{2}{3}})}{x^2} = \frac{x^{\frac{1}{3}} + 2x}{x^2} = x^{-\frac{5}{3}} + 2x^{\frac{1}{3}} = x^{-\frac{5}{3}} + 2x^{-1}. \]
      
      So, by using the power rule we get \[ g'(x) = -\frac{5}{3}x^{-\frac{5}{3}} - 2x^{-2}. \] (3 pts.)
      
      (Again, equivalent forms of final answer are acceptable and get full credit.)

   b) Find the equation of the line tangent to the graph of \( f \) at \( x = 1. \)
      
      **Solution** : We are interested in the point \((1, f(1))\) i.e. \((1, 11)\). The slope at that point is \( m = f'(1) = 1 \) (using the formula of \( f' \) that we computed above). So the equation of the tangent line at that point is : \( y - 11 = x - 1 \Rightarrow y = x + 10. \) (4 pts.)

2. (10 pts.)
   a) Compute the derivative of the function : \( h(x) = 6\sqrt{x} - \left(\frac{1}{2}\right)^x. \)
      
      **Solution** : \[ h'(x) = 6\left(\frac{1}{2}\right)^x - \left[\left(\frac{1}{2}\right)^x\right]' \Rightarrow h'(x) = 2x^{-\frac{3}{4}} - \left(\ln \frac{1}{2}\right)\left(\frac{1}{2}\right)^x. \] (5 pts.)
b) Suppose that $P(t) = 60(1.3)^t$, expresses the growth of price $P$ (in dollars) as a function of time $t$ (in years). Find the rate, in dollars per year, at which the price $P$ is increasing.

Solution: We actually want to find the derivative function $P'$. A direct differentiation gives:

$$P'(t) = 60 \left[(1.3)^t\right]' \Rightarrow P'(t) = 60(\ln 1.3)(1.3)^t \approx 15.74(1.3)^t.$$ (5 pts.)

*** END OF VERSION 2 ***