MATH 180 **Quiz 6 (Version I) 03/02/06** **Solutions 03/03/06**

Name (print) ______________________ **Discussion hour (T Th** ____**)**

1. (10 pts.) Compute the derivatives of the functions :

- $f(x) = e^{\arctan(3x^2)}$

Solution : Using the chain rule we get :

$$f'(x) = e^{\arctan(3x^2)} \cdot \left(\arctan(3x^2)\right)' = e^{\arctan(3x^2)} \cdot \frac{1}{1+(3x^2)^2} \cdot (3x^2)' = e^{\arctan(3x^2)} \cdot \frac{6x}{1+9x^4}$$

So, the answer is $\boxed{f'(x) = \frac{6x}{1+9x^4} \cdot e^{\arctan(3x^2)}}$. (**5 pts**.)

(Equivalent forms of final answer are acceptable and get full credit.)

- $g(x) = \cosh^2 x - \sinh^2 x + \ln x$

Solution : There are two ways to find the derivative function. Either use directly the chain rule to get :

$$g'(x) = 2\cosh x \cdot (\cosh x)' - 2\sinh x \cdot (\sinh x)' + (\ln x)' = 2\cosh x \sinh x - 2\sinh x \cosh x + \frac{1}{x} = \frac{1}{x}.$$

OR

Use the identity $\cosh^2 x - \sinh^2 x = 1$ to write the function as $g(x) = 1 + \ln x$. From the latter, it is easily seen that the derivative is $g'(x) = \frac{1}{x}$.

So, the answer is $\boxed{g'(x) = \frac{1}{x}}$. (**5 pts**.)

(It doesn't matter which method you used, as long as you got the correct answer.)

2. (10 pts.) Let $y = f(x)$ such that $x^3 + 2xy + y^2 = 4$.

a) Find $\frac{dy}{dx}$ using implicit differentiation.

Solution : We differentiate both sides **with respect to x** and we get :

$$\left(x^3 + 2xy + y^2\right)' = 0 \Rightarrow (x^3)' + 2(xy)' + (y^2)' = 0 \Rightarrow 3x^2 + 2\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0 \Rightarrow$$

$$\Rightarrow 3x^2 + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow (2x+2y)\frac{dy}{dx} = -3x^2 - 2y \Rightarrow \frac{dy}{dx} = -\frac{3x^2+2y}{2x+2y}$$

So, the answer is $\boxed{\frac{dy}{dx} = -\frac{3x^2+2y}{2x+2y}}$. (**6 pts**.)

b) Find the equation of the tangent line at the point $(0,2)$.

Solution : From the above formula that we computed, we can find the slope at the point $(0,2)$, which is $m = -\frac{3(0)^2+2(2)}{2(0)+2(2)} = -1$. So, the equation of the tangent line is :

$$y - 2 = -(x-0) \Rightarrow \boxed{y = -x+2}.\ \ (\textbf{4 pts}.)$$

*** END OF VERSION I ***
*** OVER FOR VERSION 2 ***

MATH 180 **Quiz 6 (Version 2) 03/02/06** **Solutions 03/03/06**

Name (print) ______________________ **Discussion hour (T Th ____)**

1. (10 pts.) Compute the derivatives of the functions :

- $f(x) = \arctan(e^{3x^2})$

  Solution : Using the chain rule we get :

$$f'(x) = \frac{1}{1+(e^{3x^2})^2} \cdot \left(e^{3x^2}\right)' = \frac{1}{1+(e^{3x^2})^2} \cdot e^{3x^2} \cdot (3x^2)' = \frac{1}{1+(e^{3x^2})^2} \cdot e^{3x^2} \cdot 6x$$

  So, the answer is $\boxed{f'(x) = \dfrac{6xe^{3x^2}}{1+e^{6x^2}}}$. (**5 pts**.)

  (Equivalent forms of final answer are acceptable and get full credit.)

- $g(x) = \cosh^2(3x) - \ln x$

  Solution : Using the chain rule and the known formulas for $\cosh x$ and $\ln x$ we get :

$$g'(x) = 2\cosh(3x) \cdot (\cosh(3x))' - (\ln x)' = 2\cosh(3x) \cdot \sinh(3x) \cdot (3x)' - \frac{1}{x} =$$

$$= 2\cosh(3x) \cdot \sinh(3x) \cdot 3 - \frac{1}{x}$$

  So, the answer is $\boxed{g'(x) = 6\cosh(3x)\sinh(3x) - \dfrac{1}{x}}$. (**5 pts**.)

2. (10 pts.) Let $y = f(x)$ such that $3x^3 + 3x^2y + y^2 = 7$.

   a) Find $\dfrac{dy}{dx}$ using implicit differentiation.

   Solution : We differentiate both sides **with respect to x** and we get :

$$\left(3x^3 + 3x^2y + y^2\right)' = 0 \Rightarrow (3x^3)' + 3(x^2y)' + (y^2)' = 0 \Rightarrow 9x^2 + 3\left(2xy + x^2\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow 9x^2 + 6xy + 3x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow (3x^2+2y)\frac{dy}{dx} = -9x^2 - 6xy \Rightarrow \frac{dy}{dx} = -\frac{9x^2+6xy}{3x^2+2y}$$

So, the answer is $\boxed{\frac{dy}{dx} = -\frac{9x^2+6xy}{3x^2+2y}}$. (**6 pts**.)

b) Find the equation of the tangent line at the point $(1,1)$.

Solution : From the above formula that we computed, we can find the slope at the point $(1,1)$, which is $m = -\frac{9(1)^2+6(1)(1)}{3(1)^2+2(1)} = -\frac{15}{5} = -3$. So, the equation of the tangent line is :

$$y - 1 = -3(x-1) \Rightarrow \boxed{y = -3x+4}.$$ (**4 pts**.)

*** END OF VERSION 2 ***