1. (10 pts.) Compute the derivatives of the functions:

- \( f(x) = e^{\arctan(3x^2)} \)

**Solution:** Using the chain rule we get:

\[
f'(x) = e^{\arctan(3x^2)} \cdot \frac{1}{1 + (3x^2)^2} \cdot 6x = \frac{6x}{1 + 9x^4} \cdot e^{\arctan(3x^2)}
\]

So, the answer is \( f'(x) = \frac{6x}{1 + 9x^4} \cdot e^{\arctan(3x^2)} \). (5 pts.)

(Equivalent forms of final answer are acceptable and get full credit.)

- \( g(x) = \cosh^2 x - \sinh^2 x + \ln x \)

**Solution:** There are two ways to find the derivative function. Either use directly the chain rule to get:

\[
g'(x) = 2 \cosh x \cdot (\cosh x)' - 2 \sinh x \cdot (\sinh x)' + (\ln x)' = 2 \cosh x \sinh x - 2 \sinh x \cosh x + \frac{1}{x} = \frac{1}{x}.
\]

**OR**

Use the identity \( \cosh^2 x - \sinh^2 x = 1 \) to write the function as \( g(x) = 1 + \ln x \). From the latter, it is easily seen that the derivative is \( g'(x) = \frac{1}{x} \).

So, the answer is \( g'(x) = \frac{1}{x} \). (5 pts.)

(It doesn't matter which method you used, as long as you got the correct answer.)

2. (10 pts.) Let \( y = f(x) \) such that \( x^3 + 2xy + y^2 = 4 \).

a) Find \( \frac{dy}{dx} \) using implicit differentiation.

**Solution:** We differentiate both sides with respect to \( x \) and we get:

\[
(x^3 + 2xy + y^2)' = 0 \Rightarrow (x^3)' + 2(xy)' + (y^2)' = 0 \Rightarrow 3x^2 + 2 \left( y + x \frac{dy}{dx} \right) + 2y \frac{dy}{dx} = 0 \Rightarrow
\]
\[3x^2 + 2y + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \Rightarrow (2x + 2y) \frac{dy}{dx} = -3x^2 - 2y \Rightarrow \frac{dy}{dx} = \frac{-3x^2 - 2y}{2x + 2y}\]

So, the answer is \(\frac{dy}{dx} = \frac{-3x^2 + 2y}{2x + 2y}\). (6 pts.)

b) Find the equation of the tangent line at the point \((0, 2)\).

Solution: From the above formula that we computed, we can find the slope at the point \((0, 2)\), which is \(m = \frac{-3(0)^2 + 2(2)}{2(0) + 2(2)} = -1\). So, the equation of the tangent line is:

\[y - 2 = -(x - 0) \Rightarrow y = -x + 2\] (4 pts.)
1. (10 pts.) Compute the derivatives of the functions:

- \( f(x) = \arctan(e^{3x^2}) \)

**Solution:** Using the chain rule we get:

\[
f'(x) = \frac{1}{1 + (e^{3x^2})^2} \cdot (e^{3x^2})' = \frac{1}{1 + (e^{3x^2})^2} \cdot e^{3x^2} \cdot (3x^2)' = \frac{1}{1 + (e^{3x^2})^2} \cdot e^{3x^2} \cdot 6x
\]

So, the answer is \( f'(x) = \frac{6xe^{3x^2}}{1 + e^{6x^2}} \). (5 pts.)

(Equivalent forms of final answer are acceptable and get full credit.)

- \( g(x) = \cosh^2(3x) - \ln x \)

**Solution:** Using the chain rule and the known formulas for \( \cosh x \) and \( \ln x \) we get:

\[
g'(x) = 2 \cosh(3x) \cdot (\cosh(3x))' - (\ln x)' = 2 \cosh(3x) \cdot \sinh(3x) \cdot (3x)' - \frac{1}{x} = \]

\[
= 2 \cosh(3x) \cdot \sinh(3x) \cdot 3 - \frac{1}{x}
\]

So, the answer is \( g'(x) = 6 \cosh(3x) \sinh(3x) - \frac{1}{x} \). (5 pts.)

2. (10 pts.) Let \( y = f(x) \) such that \( 3x^3 + 3x^2y + y^2 = 7 \).

a) Find \( \frac{dy}{dx} \) using implicit differentiation.

**Solution:** We differentiate both sides with respect to \( x \) and we get:

\[
(3x^3 + 3x^2y + y^2)' = 0 \Rightarrow (3x^3)' + 3(x^2y)' + (y^2)' = 0 \Rightarrow 9x^2 + 3(2xy + x^2 \frac{dy}{dx}) + 2y \frac{dy}{dx} = 0
\]
\[ 9x^2 + 6xy + 3x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \Rightarrow (3x^2 + 2y) \frac{dy}{dx} = -9x^2 - 6xy \Rightarrow \frac{dy}{dx} = \frac{-9x^2 - 6xy}{3x^2 + 2y} \]

So, the answer is \[ \frac{dy}{dx} = \frac{-9x^2 - 6xy}{3x^2 + 2y} \] (6 pts.)

b) Find the equation of the tangent line at the point \((1, 1)\).

**Solution**: From the above formula that we computed, we can find the slope at the point \((1, 1)\), which is \( m = -\frac{9(1)^2 + 6(1)(1)}{3(1)^2 + 2(1)} = -\frac{15}{5} = -3 \). So, the equation of the tangent line is:

\[ y - 1 = -3(x - 1) \Rightarrow y = -3x + 4. \] (4 pts.)

*** END OF VERSION 2 ***