1. (8 pts.) Let \( f(x) = x^3 - 3x^2 + 3ax - 17 \), where \( a \) is a fixed real number.
   
   (a) Find the number of critical points of \( f(x) \) when \( a = 1 \).
   
   **Solution:** \( f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x - 1)^2 \) so \( f'(x) = 3(x - 1)^2 \) (2 points). Therefore \( f(x) \) has one critical point (2 points), at \( x = 1 \).

   (b) For any fixed value of \( a \), find all of the inflection points on the graph of \( y = f(x) \).
   
   **Solution:** \( f''(x) = 6x - 6 = 6(x - 1) \) (1 point). Since the concavity goes from down to up as \( x \) passes through \( x = 1 \), there is one inflection point \( (1, 3a - 19) \) (3 points).

2. (12 pts.) Let \( f(x) = 16x + \frac{9}{x} \), where \( x > 0 \).
   
   (a) Find the intervals on which \( f(x) \) is increasing.
   
   **Solution:** \( f'(x) = 16 - \frac{9}{x^2} = \frac{16x^2 - 9}{x^2} \) (2 points) which means there is only one critical point which is at \( x = \frac{3}{4} \). Since \( f'(\frac{1}{2}) < 0 \) and \( f'(1) > 0 \), the function \( f(x) \) is increasing on \( \left[ \frac{3}{4}, \infty \right) \) (3 points).

   (b) Find the intervals on which \( f(x) \) is decreasing.
   
   **Solution:** Using the calculations of part (a) we see that the function is decreasing on \( (0, \frac{3}{4}) \) (3 points)

   (c) Find all local minima for \( f(x) \). Justify your answer by the second derivative test.
   
   **Solution:** There is only one critical point at \( x = \frac{3}{4} \). Since \( f''(x) = \frac{18}{x^3} \) it follows that \( f''(\frac{3}{4}) > 0 \) and thus \( \left( \frac{3}{4}, 24 \right) \) is a local minimum (4 points).
1. (8 pts.) Let \( f(x) = x^3 + 6x^2 + 3ax + 5 \), where \( a \) is a fixed real number.

(a) Find the number of critical points of \( f(x) \) when \( a = 0 \).

Solution: \( f'(x) = 3x^2 + 12x = 3x(x+4) \) so \( f'(x) = 3x(x+4) \) (2 points). Therefore \( f(x) \) has two critical points (2 points), at \( x = 0, -4 \).

(b) For any fixed value of \( a \), find all of the inflection points on the graph of \( y = f(x) \).

Solution: \( f''(x) = 6x + 12 = 6(x+2) \) (1 point). Since the concavity goes from down to up as \( x \) passes through \( x = -2 \), there is one inflection point \((-2, 21 - 6a)\) (3 points).

2. (12 pts.) Let \( f(x) = 25x + \frac{4}{x} \), where \( x > 0 \).

(a) Find the intervals on which \( f(x) \) is increasing.

Solution: \( f'(x) = 25 - \frac{4}{x^2} = \frac{25x^2 - 4}{x^2} \) (2 points) which means there is only one critical point which is at \( x = \frac{2}{5} \). Since \( f'(\frac{1}{5}) < 0 \) and \( f'(1) > 0 \), the function \( f(x) \) is increasing on \( \left[ \frac{2}{5}, \infty \right) \) (3 points).

(b) Find the intervals on which \( f(x) \) is decreasing.

Solution: Using the calculations of part (a) we see that the function is decreasing on \( (0, \frac{2}{5}] \) (3 points).

(c) Find all local minima for \( f(x) \). Justify your answer by the second derivative test.

Solution: There is only one critical point at \( x = \frac{2}{5} \). Since \( f''(x) = \frac{8}{x^3} \) it follows that \( f''(\frac{2}{5}) > 0 \) and thus \( \left( \frac{2}{5}, 20 \right) \) is a local minimum (4 points).