MATH 180 Final Examination **Thursday, May 4, 2006**

SOLUTION

Boldface indicates point value.

1. (30)

(a) $\boxed{(\ln 3)3^x + 3x^2 + \frac{1}{3} - \frac{3}{x^2} \quad \textbf{2, 1, 1, 2 for summands}}$

(b) $\boxed{\frac{(4x^3 + \sin x)(2x^6 + \tan x) - (x^4 - \cos x)(12x^5 + \sec^2 x)}{(2x^6 + \tan x)^2} \quad \textbf{numerator 4, denominator 2}}$

(c) $\boxed{[\frac{1}{2}((\ln x)^2 + 5)^{-1/2}](2\ln x)(\frac{1}{x}) \quad \textbf{2, 2, 2, for factors}}$

(d) $\boxed{(231(x^2+1)^{230}(2x))(1 + \sin x) + (x^2+1)^{231}(\cos x) \quad \textbf{4, 2 for summands}}$

(e) $\boxed{3e^{3x} + 4\cosh 4x + \frac{1}{1+x^2} - \frac{1}{2}x^{-3/2} \quad \textbf{2, 1, 1, 2 for summands}}$

2. (15)

$$\begin{aligned}
f'(x) &= \lim_{x \longrightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \textbf{3} \\
&= \lim_{x \longrightarrow 0} \frac{[2(x+h)^2 - (x+h) - 1] - [2x^2 - x - 1]}{h} \\
&= \lim_{x \longrightarrow 0} \frac{[2(x^2 + 2xh + h^2) - (x+h) - 1] - [2x^2 - x - 1]}{h} \\
&= \lim_{x \longrightarrow 0} \frac{4xh + 2h^2 - h}{h} \quad \textbf{9 for calculations} \\
&= \lim_{x \longrightarrow 0} 4x + 2h - 1 \\
&= 4x - 1 \quad \textbf{3 for last two lines}
\end{aligned}$$

3. (20)

(a) $\boxed{v(t) = s'(t) = -12t + 3 \; meters/second \quad \mathbf{3}}$ and $\boxed{a(t) = v'(t) = -12 \; meters/second^2 \quad \mathbf{2}}$.

(b) $\boxed{\text{When } 0 = v(t) = -12t + 3 \quad \mathbf{2}}$; thus $\boxed{t = \frac{1}{4} \quad \mathbf{3}}$.

(c) $\boxed{s(\frac{1}{4}) \quad \mathbf{2}} = -6(\frac{1}{16}) + 3(\frac{1}{4}) + 30 = \boxed{\frac{243}{8} = 30.375 \quad \mathbf{3}}$

(d) When $\boxed{s(t) = 0 \quad \mathbf{2}}$, or equivalently $2t^2 - t - 10 = 0$, $(2t - 5)(t + 2) = 0$. Thus $\boxed{t = \frac{5}{2} \text{ seconds} \quad \mathbf{3}}$

4. (20)

(a) $f'(x)$ changes sign from negative to positive at $\boxed{x = -3 \quad \mathbf{4} \text{ and } x = 3 \quad \mathbf{4}}$.

(b) $f'(x)$ is decreasing on $[-2, 2]$ Thus the graph of $y = f(x)$ is concave down on $\boxed{(-2, 2) \quad \mathbf{4}}$.

(c) $f'(x)$ is increasing on $[-4, -2]$ and $[2, 4]$ Thus the graph of $y = f(x)$ is concave up on $(-4, -2)$ and $(2, 4)$ In light of part (c) $\boxed{x = -2 \quad \mathbf{4} \text{ and } x = 2 \quad \mathbf{4}}$.

5. (20) Since $y^3x^2 + xy - x^2 = 2$,

(a) $[(3y^2y')x^2 + y^3(2x)] + [(1)y + x(y')] - 2x = 0$. Thus $\boxed{y' = \frac{2x - y - 2xy^3}{3x^2y^2 + x} \quad \mathbf{6}}$ and at $(2, 1)$ $\boxed{y' = -\frac{1}{14} \approx -.0714 \quad \mathbf{4}}$. An equation of the tangent line is $\boxed{y - 1 = -\frac{1}{14}(x - 2) \quad \mathbf{4}}$

(b) $f(2.1) \approx \boxed{-\frac{1}{14}(2.1 - 2) + 1 = -\frac{1}{14}(\frac{1}{10}) + 1 = \frac{139}{140} \approx .9929 \quad \mathbf{6}}$

6. (25) First of all

$$\begin{aligned} f(x) &= x^4 + x^3 &&= x^3(x + 1) \\ f'(x) &= 4x^3 + 3x^2 &&= x^2(4x + 3) \\ f''(x) &= 12x^2 + 6x &&= 6x(2x + 1) \end{aligned}$$

(a) Solve $f'(x) = 0$. $\boxed{x = 0 \quad \mathbf{2} \text{ and } x = -\frac{3}{4} \quad \mathbf{2}}$.

(b) Note $f'(-1) < 0$, $f'(-0.5) > 0$, and $f'(1) > 0$. In light of part (a) $\boxed{increasing\ [-\frac{3}{4}, \infty) \quad \mathbf{2}}$ and $\boxed{decreasing\ (-\infty, -\frac{3}{4}] \quad \mathbf{2}}$.

(c) Note $f''(x) = 0$ holds if and only if $x = 0, -\frac{1}{2}$; $f''(-1) > 0$, $f''(-0.25) < 0$, and $f''(1) > 0$. Thus $\boxed{\textit{concave down } (-\frac{1}{2}, 0) \quad \mathbf{2}}$ and $\boxed{\textit{concave up } (-\infty, -\frac{1}{2}), (0, \infty) \quad \mathbf{2}}$.

(d) In light of part (c) the inflection points are $\boxed{(-\frac{1}{2}, -\frac{1}{16}) \quad \mathbf{3}, (0,0) \quad \mathbf{1}}$.

(e) $\boxed{\text{Shape of graph } \mathbf{5}}$, $\boxed{\text{points plotted } \mathbf{4}}$. In particular, local minimum $(-\frac{3}{4}, -\frac{27}{256})$, no local maximum, the graph crosses the x-axis at $(-1, 0)$ and $(0, 0)$. For a sketch of the graph, see "FinS06 Graph".

7. (15) Find:

(a) $\boxed{\lim_{x \longrightarrow 0} \frac{e^{3x} - 1 - 3x}{x^2} = \lim_{x \longrightarrow 0} \frac{3e^{3x} - 3}{2x} = \lim_{x \longrightarrow 0} \frac{9e^{3x}}{2} = \frac{9}{2} \quad \mathbf{8}}$

(b) $\lim_{x \longrightarrow 3^-} f(x) = \lim_{x \longrightarrow 3^-} x^2 + 4x = 21$ and $\lim_{x \longrightarrow 3^+} f(x) = \lim_{x \longrightarrow 3^-} 2cx + 1 = 6c + 1$. Thus $\boxed{21 = 6c + 1 \quad \mathbf{3}}$ and $\boxed{c = \frac{10}{3} \quad \mathbf{4}}$.

8. (15) Let $f(x) = x^2 + x$.

(a) $\Delta x = \frac{2 - 0}{4} = \frac{1}{2}$. Thus

$$\begin{aligned} \textit{r.h.sum} &= \boxed{\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right)(\tfrac{1}{2}) \quad \mathbf{5}} \\ &= \left(\left(\frac{1}{4} + \frac{1}{2}\right) + (1 + 1) + \left(\frac{9}{4} + \frac{3}{2}\right) + (4 + 2)\right)\left(\frac{1}{2}\right) \\ &= \boxed{\frac{25}{4} \quad \mathbf{5}} \end{aligned}$$

(b) $\boxed{\text{An overestimate since } f(x) \text{ is increasing on } [0, 2] \quad \mathbf{5}}$. Indeed, all right hand sums are overestimates for the same reason. One can also use the Fundamental Theorem of Calculus to compute the definite integral and compare it with the estimate.

9. (20)

(a) Let $F(x) = \frac{x^4}{4}$. Then $F'(x) = x^3$.

$$\boxed{\frac{1}{3 - (-1)} \int_{-1}^{3} x^3 \, dx \ \mathbf{4}} = \boxed{\frac{1}{4}(F(3) - F(-1)) \quad \mathbf{3}} = \frac{1}{4}\left(\frac{81}{4} - \frac{1}{4}\right) = \boxed{5 \quad \mathbf{3}}$$

(b) Let $F(x) = e^x + 2x$. Then $F'(x) = e^x + 2$.

$$\boxed{\int_0^{\ln 2} (e^x + 2)\,dx \ \mathbf{3}} = \boxed{F(\ln 2) - F(0) \quad \mathbf{3}}$$

$$= (e^{\ln 2} + 2\ln 2) - (e^0 + 0) = (2 + 2\ln 2) - 1 = \boxed{2\ln 2 + 1 \quad \mathbf{4}}$$

10. (20)

(a) $5,000 = xy$ which means $y = \dfrac{5,000}{x}$. Therefore $C(x) = 3(25x) + 2(12y) = \boxed{75x + \dfrac{120,000}{x} \quad \mathbf{5}}$ and has domain the set of all $\boxed{x > 0 \quad \mathbf{3}}$.

(b) $0 = C'(x) = 75 - \dfrac{120,000}{x^2}$ means $\boxed{x = \sqrt{\dfrac{120,000}{75}} = 40 \quad \mathbf{4}}$.

Since $C'(1) < 0$ and $C'(120,000) > 0$, $C(x)$ is decreasing on $(0, 40)$ and increasing on $(40, \infty)$. Therefore $C(x)$ has a minimum at $x = 40$. **4**

(c) $\boxed{C(40) = \$6,000 \quad \mathbf{4}}$