MATH 180 **Quiz 2** **01/26/06**

Name (print) : **Discussion Hour : 11 12 1**

1. (10 pts.)

   a) Find k so that the following function is continuous on *any* interval :

$$f(x) = \begin{cases} 3x & , \quad x \leq 2 \\ kx^2 - 6 & , \quad x > 2 \end{cases}$$

   Solution : We need to check continuity at $x = 2$. We have :

   $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 3x = 6$. Moreover, $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (kx^2 - 6) = 4k - 6$.

   So, in order for the limit to exist, we must have that $4k - 6 = 6$, which gives $k = 3$. Now, for $k = 3$, we also have that $\lim_{x \to 2} f(x) = f(2) = 6$. So, the function is continuous at $x = 2$ and therefore it is continuous on any interval. Hence, the answer is $\boxed{k = 3}$.

   b) Let $g(x) = 2 \sin x + 3 \cos x$. Show that there exists a number c, with $0 \leq c \leq \pi$, such that $g(c) = 0$.

   Solution : The given function is continuous on the interval $[0, \pi]$, since it is a sum of continuous functions. We also have that:

$$g(0) = 2 \sin 0 + 3 \cos 0 = 0 + 3 = 3$$
$$g(\pi) = 2 \sin \pi + 3 \cos \pi = 0 - 3 = -3.$$

   So, for $k = 0$ which lies between $g(0)$ and $g(\pi)$, by the Int. Value Thm, there exists a number c in the interval $[0, \pi]$ (i.e. $0 \leq c \leq \pi$), such that $g(c) = k = 0$.

2. (10 pts.) Let $f(x) = \dfrac{x^2 + 4x + k}{x + 2}$.

   a) Find k such that $\lim_{x \to -2} f(x)$ exists.

   Solution : We notice that for the denominator $\lim_{x \to -2} (x + 2) = 0$. Therefore, the limit of the given function can only exist if the same is true for the numerator i.e.

$$\lim_{x \to -2} (x^2 + 4x + k) = 0 \Rightarrow 4 - 8 + k = 0 \Rightarrow \boxed{k = 4}.$$

   b) Is f continuous on the interval $[-\pi, 1]$?

   Solution : The function is not continuous on the interval $[-\pi, 1]$, since the latter contains the root of the denominator.

   c) Compute $\lim_{x \to 0} f(x)$.

   Solution : We have the following:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + 4x + 4}{x + 2} = \frac{\lim_{x \to 0} (x^2 + 4x + 4)}{\lim_{x \to 0} (x + 2)} = \frac{0 + 0 + 4}{0 + 2} = \frac{4}{2} = 2.$$