1. (10 pts.)
   a) Find \( k \) so that the following function is continuous on *any* interval:
   \[
   f(x) = \begin{cases} 
   3x & , \quad x \leq 2 \\
   kx^2 - 6 & , \quad x > 2
   \end{cases}
   \]
   **Solution**: We need to check continuity at \( x = 2 \). We have:
   \[
   \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 3x = 6.
   \]
   Moreover,
   \[
   \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (kx^2 - 6) = 4k - 6.
   \]
   So, in order for the limit to exist, we must have that \( 4k - 6 = 6 \), which gives \( k = 3 \). Now, for \( k = 3 \), we also have that \( \lim_{x \to 2^-} f(x) = f(2) = 6 \).
   So, the function is continuous at \( x = 2 \) and therefore it is continuous on any interval. Hence, the answer is \( k = 3 \).

   b) Let \( g(x) = 2 \sin x + 3 \cos x \). Show that there exists a number \( c \), with \( 0 \leq c \leq \pi \), such that \( g(c) = 0 \).
   **Solution**: The given function is continuous on the interval \([0, \pi]\), since it is a sum of continuous functions. We also have that:
   \[
   g(0) = 2 \sin 0 + 3 \cos 0 = 0 + 3 = 3
   \]
   \[
   g(\pi) = 2 \sin \pi + 3 \cos \pi = 0 - 3 = -3.
   \]
   So, for \( k = 0 \) which lies between \( g(0) \) and \( g(\pi) \), by the Int. Value Thm, there exists a number \( c \) in the interval \([0, \pi]\) (i.e. \( 0 \leq c \leq \pi \)), such that \( g(c) = k = 0 \).

2. (10 pts.) Let \( f(x) = \frac{x^2 + 4x + k}{x + 2} \).
   a) Find \( k \) such that \( \lim_{x \to -2} f(x) \) exists.
   **Solution**: We notice that for the denominator \( \lim_{x \to -2} (x + 2) = 0 \).
   Therefore, the limit of the given function can only exist if the same is true for the numerator i.e.
   \[
   \lim_{x \to -2} (x^2 + 4x + k) = 0 \Rightarrow 4 - 8 + k = 0 \Rightarrow k = 4.
   \]
   b) Is \( f \) continuous on the interval \([-\pi, 1]\) ?
   **Solution**: The function is not continuous on the interval \([-\pi, 1]\), since the latter contains the root of the denominator.
   c) Compute \( \lim_{x \to 0} f(x) \).
   **Solution**: We have the following:
   \[
   \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + 4x + 4}{x + 2} = \frac{\lim_{x \to 0} (x^2 + 4x + 4)}{\lim_{x \to 0} (x + 2)} = \frac{0 + 0 + 4}{0 + 2} = \frac{4}{2} = 2.
   \]