MATH 425 # Hour Exam II **Radford 04/17/2009**

Name (print) ________________________________

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. *Answers must be justified.* (4) There are *four questions* on this exam. (5) Each question counts 25 points. (6) *You are expected to abide by the University's rules concerning academic honesty.*

You may paraphrase theorems in the book; for example "groups of order p^2, where p is prime, are abelian".

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1. Let $F = \mathbf{Q}(\sqrt{5} + \imath\sqrt{3})$.

(a) Show that $F = \mathbf{Q}(\sqrt{5}, \imath\sqrt{3})$ and $[F : \mathbf{Q}] = 4$.

(b) Find the minimal polynomial of $\alpha = \sqrt{5} + \imath\sqrt{3}$ over $\mathbf{Q}$.

(c) Find the minimal polynomial of $\alpha = \sqrt{5} + \imath\sqrt{3}$ over $\mathbf{Q}(\sqrt{5})$.

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2. Let $F = \mathbf{Q}(3^{1/2}, 7^{1/3})$.

(a) Find $[F : \mathbf{Q}]$.

(b) Find a basis for F as a vector space over $\mathbf{Q}$.

(c) Show that $f(x) = x^4 - 22x^3 + 6x + 6$ has no root in F.

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3. Suppose G is a finite group and $|G| = 135 = 3^3{\cdot}5$.

(a) Show that G has a subgroup of order 45.

(b) Show that G has a subgroup of order 15.

(c) Show that G has an element of order 15.

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4. Find:

(a) A splitting field of $x^4 - 7$ over $\mathbf{Q}$, and

(b) A presentation of the multiplicative group $G = G_1{\times}G_2$, where G_1 is cyclic of order 2 and G_2 is cyclic of order $n \geq 2$. (No justification necessary.)