1. Page 540, number 2: (30 points) 2 (10), 3 (10), 3 (10) respectively.

2. Page 540, number 4: (30 points) Write $u = (a_1, \ldots, a_n)$ and $v = (b_1, \ldots, b_n)$. Then

$$d(u, v) = |\{i \mid 1 \leq i \leq n, a_i \neq b_i\}|$$

and thus

$$d(u, v) = n - |\{i \mid 1 \leq i \leq n, a_i = b_i\}|. \quad (1)$$

(a) For $1 \leq i \leq n$, $a_i = b_i$ if and only if $b_i = a_i$. Therefore $d(u, v) = d(v, u)$ by (1) (10).

(b) By (1) observe that $d(u, v) = 0$ if and only if $|\{i \mid 1 \leq i \leq n, a_i = b_i\}| = n$ if and only if $a_i = b_i$ for all $1 \leq i \leq n$ if and only if $u = v$ (10).

(c) Write $w = (c_1, \ldots, c_n)$. Then $u + w = (a_1 + c_1, \ldots, a_n + c_n)$ and $v + w = (b_1 + c_1, \ldots, b_n + c_n)$. Let $1 \leq i \leq n$. Since $a_i + c_i = b_i + c_i$ if and only if $a_i = b_i$, $d(u + w, v + w) = d(u, v)$ by (1) again (10).

3. Page 541, number 14: (40 points) Let $F = \mathbb{Z}_2$ and $V \subseteq F^n$ be the binary code. Fix $1 \leq i \leq n$. Then the map $\pi : V \longrightarrow F$ defined by $\pi(a_1, \ldots, a_n) = a_i$ is a homomorphism of additive groups. Note that

$$\text{Ker}\pi = \{(a_1, \ldots, a_n) \in V \mid a_i = 0\}$$

and thus consists of the elements of $V$ whose $i^{th}$ component is 0 (10).

Suppose that $\text{Im}\pi = (0)$. Then $\text{Ker}\pi = V$ which means that all code words (elements of $V$) have 0 in the $i^{th}$ component (10).

Suppose that $\text{Im}\pi \neq (0)$. Then $\text{Im}\pi = F$ as the latter has 2 elements. Since $V/\text{Ker}\pi \cong F$ by the First Isomorphism Theorem for groups (10), the calculation $|V| = [V : \text{Ker}\pi]|\text{Ker}\pi| = 2|\text{Ker}\pi|$ shows that half of the element of $V$ have 0 in the $i^{th}$ component (10).