MATH 180 — Hour Exam II Solution — Radford — 04/09/02

1. (20 points):

a) $21x^2 - (12/x^3)$ (5 points);

b) $200(x^2-7)^{199}(2x)(x^4-7x+1) + (x^2-7)^{200}(4x^3-7)$ (5 points);

c) $[(\cos x + 10x^9)(3^x+1) - (\sin x + x^{10} - 3)(3^x \ln 3)]/(3^x+1)^2$ (5 points);

d) $e^{\cosh x}\sinh x - (\sin x/\cos x)$ (5 points).

2. (15 points): a) Since $x^3y^4 + 4y^5 + 2x^7 = 1$, $(3x^2y^4 + x^3 4y^3 y') + 4(5y^4 y') + 2(7x^6) = 0$ (4 points) and thus $dy/dx = y' = -(3x^2y^4 + 14x^6)/(4x^3y^3 + 20y^4)$ (4 points). b) When $x = -1$ and $y = 1$, $dy/dx = -17/16$ (3 points). Thus $y - 1 = -(17/16)(x+1) = -(17/16)x - 1/16$ (4 points).

3. (18 points): $f(x) = x^4 - 8x^2 = x^2(x^2-8)$.

a) $f'(x) = 4x^3 - 16x = 4x(x^2-4)$. The critical points are $x = -2, 0, 2$ (3 points).

b) $f'(-10) < 0, f'(-1) > 0, f'(1) < 0, f'(10) > 0$. Thus $f(x)$ is increasing on $(-2,0), (2,\infty)$, $f(x)$ is decreasing on $(-\infty,-2), (0,2)$ (3 points).

c) $f''(x) = 12x^2 - 16 = 12(x^2 - 4/3)$. Thus $f''(x) = 0$ has solutions $x = -2/\sqrt{3}, 2/\sqrt{3}$. $f''(-10) > 0, f''(0) < 0, f'''(10) > 0$. The graph $y = f(x)$ is concave up on $(-\infty, -2/\sqrt{3}), (2/\sqrt{3}, \infty)$ and concave down on $(-2/\sqrt{3}, 2/\sqrt{3})$ (3 points).

d) Inflection points $(-2/\sqrt{3}, -80/9), (2/\sqrt{3}, -80/9)$ (3 points).

e) Correct shape (3 points); Plot the inflection points $(-2/\sqrt{3}, -80/9), (2/\sqrt{3}, -80/9)$, the points corresponding to local maxima and local minima $(-2,-16), (0,0,), (2,-16)$, where the graph crosses the x-axis $(-2\sqrt{2}, 0), (0,0), (2\sqrt{2}, 0)$ (3 points).

4. (17 points): a) $C(x) = 2(25y) + 10x = 50y + 10x = 50(500/x) + 10x = 10((2500/x) + x)$, domain $0 < x$ (5 points). b) $C'(x) = 10(-2500/x^2 + 1) = 10(-2500 + x^2)/x^2 = 0$ has solution(s) $x = 50$ (4 points). Note that $C'(1) < 0$ and $C'(60) > 0$. Therefore $C(x)$ has a minimum at $x = 50$ (4 points). Thus $x = 50, y = 10$ (4 points).

5. (15 points): a) $\lim_{x\to\infty}(7x^2+3x-1)/(2x^2+4) = \lim_{x\to\infty}(14x+3)/(4x) = \lim_{x\to\infty}(14/4) = 14/4 = 7/2$ (8 points); b) $\lim_{x\to 0}(\sin 3x)/4x = \lim_{x\to 0}(3\cos 3x)/4 = 3/4$ (7 points).

6. (15 points): $f(x) = \sqrt{x^2+5}$. Thus $f'(x) = x/\sqrt{x^2+5}$ and

a) $f'(2) = 2/3$ (5 points), $f(x) \approx f'(2)(x-2) + f(2) = (2/3)(x-2) + 3$ (5 points);

b) $f(2.1) \approx (2/3)(2.1-2) + 3 = 1/15 + 3 = 46/15$ (5 points).