1. (15 points) \( f(x) = \begin{cases} \frac{kx + 8}{x^2 - 4} & : x \leq -2 \\ \frac{kx^2 - 4}{x + 2} & : x > -2 \end{cases} \).

a) The value of \( k \) we seek is the one which makes the one-sided limits \( \lim_{x \to -2^-} f(x) = k(-2) + 8 \) and \( \lim_{x \to -2^+} f(x) = k(-2)^2 - 4 = 4k - 4 \) equal. Thus \(-2k + 8 = 4k - 4 \) (4pts), or \( k = 2 \) (4pts).

b) Graph of \( y = -2x + 8 \), where \( x \leq -2 \) (3pts), together with graph of \( y = 4x^2 - 8 \), where \( x \geq -2 \) (4pts).

2. (15 points) \( f(x) = 3x^2 - x \). Thus

\[
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{[3(a+h)^2 - (a+h)] - [3a^2 - a]}{h} = \lim_{h \to 0} \frac{3(2ah + h^2) - h}{h} = \lim_{h \to 0} 6a + 3h - 1 \text{ (calculations, 9pts)} = 6a - 1 \text{ (1pt)}.
\]

3. (15 points) \( x^2 + 3xy + 3y^2 = 7 \). Differentiating both sides with respect to \( x \) yields

a) \( 2x + 3(y + x \frac{dy}{dx}) + 3(2y \frac{dy}{dx}) = 0 \) (4pts), and thus \( \frac{dy}{dx} = -\frac{2x + 3y}{3x + 6y} \) (4pts).

b) \( \frac{dy}{dx} = -\frac{5}{9} \) (4pts) when \((x, y) = (1, 1)\). Thus \( y - 1 = -\frac{5}{9}(x - 1) \), or \( y = -\frac{5}{9}x + \frac{14}{9} \) (3pts).

4. (15 points) a) B, G; or B; or G   (3pts) b) D, E   (3pts) c) D   (3pts) d) A   (3pts) e) A, E, F   (3pts)

5. (30 points) Answers unsimplified. **Point value for parts in boldface.**

a) \((x^3 - \frac{7}{x} + 6)^{298} \ln(\cos x)\)' = 
\[298(x^3 - \frac{7}{x} + 6)^{297}(2)(3x^2 - \frac{7}{x^2})(2)\ln(\cos x)(1) + (x^3 - \frac{7}{x} + 6)^{298}(2)[\frac{1}{\cos x}](2)(-\sin x)(1)\].

b) \( \left(\frac{7x}{\sqrt{x} - 6}\right)' = \left[\frac{7x \ln 7(2)(\sqrt{x} - 6)(2) - 7(2)^2(2)}{(\sqrt{x} - 6)^2(2)}\right] \cdot \] 

\( c) (e^{9x^3 + \pi x} \sinh(2x))' = [e^{9x^3 + \pi x}(2)(-27x^2 + \pi)(2)]\sinh(2x)(1) + e^{9x^3 + \pi x}(2)[(\cosh(2x))(2)(2)(1)] \).
6. (20 points) \( f(x) = x^4 - 3x^2;\ f'(x) = 4x^3 - 6x = 2x(2x^2 - 3),\ f''(x) = 12x^2 - 6 = 6(2x^2 - 1).\)

   a) \( x \)-coordinates of the critical points: \( x = 0 \) (2pts), \(-\sqrt{3}/2\) (2pts), \(\sqrt{3}/2\) (2pts).

   b) \[
   \begin{array}{c|c|c|c|c|c}
   \text{dec} & \text{inc} & \text{dec} & \text{inc} & f(x) \\
   \hline
   (-) & -\sqrt{3}/2 & (+) & 0 & \sqrt{3}/2 & (+) & f'(x) \\
   \end{array}
   \]

   derived from test values. \( f(x) \) is increasing on \([-\sqrt{3}/2, 0]\) (1pt), \([\sqrt{3}/2, \infty)\) (1pt); \( f(x) \) is decreasing on \((-\infty, -\sqrt{3}/2);\) (1pt), \([0, \sqrt{3}/2]\) (1pt).

   c) \[
   \begin{array}{c|c|c|c|c|c|c}
   \text{concave up} & \text{concave down} & \text{concave up} & f(x) \\
   \hline
   (+) & -1/2 & (--) & \sqrt{1/2} & (+) & f''(x) \\
   \end{array}
   \]

   derived from test values. Graph of \( f(x) \) is concave up on \((-\infty, -\sqrt{1/2})\) (2pts), \((\sqrt{1/2}, \infty)\) (2pts); down on \((-\sqrt{1/2}, \sqrt{1/2})\) (2pts).

   d) Inflection points: \((-\sqrt{1/2}, -5/4)\) (2pts), \((\sqrt{1/2}, -5/4)\) (2pts).

7. (10 points) L'Hopital's rule does apply and \[
\lim_{x \to 0} \frac{e^x - 1}{\sin 3x} = \lim_{x \to 0} \frac{e^x}{3 \cos 3x} = \frac{1}{3} (5\text{pts}).
\]

8. (12 points) Since \( V = \frac{4}{3}\pi r^3,\ \frac{dV}{dt} = \frac{4}{3}\pi (3r^2 \frac{dr}{dt}) = 4\pi r^2 \frac{dr}{dt} (6\text{pts}).\) Given that \( r = 11 \) inches and \( \frac{dr}{dt} = .5 \) (1pt) inches/second, we have \[
\frac{dV}{dt} = 4\pi 11^2 (.5) = 242\pi \text{ cubic inches/second (2pts).}
\]

9. (13 points) \( f(x) = (x^2 + 9)^{3/2}.\)

   a) \( f'(x) = \frac{3}{2}(x^2 + 9)^{1/2}(2x) = (x^2 + 9)^{1/2}3x \) (4pts); thus \( f'(4) = 60.\) \( y = f'(4)(x - 4) + f(4) = 60(x - 4) + 125;\) \( f(x) \approx 60(x - 4) + 125 \) (4pts).

   b) \( f(4.1) \approx 60(4.1 - 4) + 125 = 6 + 125 = 131 \) (5pts).

10. (20 points) \( y = f(x) = 4x^3 + 2.\)

   a) Area = \( \int_1^3 (4x^3 + 2) \, dx \) (5pts) = \( \int_1^3 (x^4 + 2x) \, dx \) (5pts) = \([3^4 + 2(3)] - [1^4 + 2(1)] = 84 \) (5pts).

   b) Average value = \( \frac{84}{(3 - 1)} = 42 \) (5pts).

11. (15 points) \( \Delta x = (6 - 0)/3 = 2.\) \( LHS = (2^0 + 2^2 + 2^4)2 = 42 \) (3pts), \( RHS = (2^2 + 2^4 + 2^6)2 = 168 \) (4pts). Numerical answers only, 3 points total.

12. (20 points) Garden: 1,000 square foot rectangular garden plot \( x \) feet long and \( y \) feet wide. Fencing for two lengths \( x \) feet, $5 per foot, for three lengths \( y \) feet, $10 per foot.

   a) Cost function \( C = C(x) = 2(5x) + 3(10y) = 10x + 30000 \) (6pts), domain \( 0 < x \) (2pts).

   b) \( C'(x) = 10 - \frac{30000}{x^2} = 10 \frac{x^2 - 3000}{x^2}.\) Thus \( C'(x) = 0 \) if and only if \( x = \sqrt{3000} \) (4pts).

   Since \[
   \begin{array}{c|c|c|c|c|c|c}
   \text{dec} & \text{inc} & f(x) & f'(x) \\
   \hline
   0 & -\sqrt{3000} & + & 0 & - & \sqrt{3000} & + \\
   \end{array}
   \]

   it follows that \( C(\sqrt{3000}) \) is a minimum for the function \( C(x) \) (4pts). Thus \( x = \sqrt{3000}, y = \frac{1000}{\sqrt{3000}} \) are the dimensions (4pts).