MATH 180  Hour Exam I  Radford  10/01/04

Name (print) ______________________________ Tu/Th Discussion (circle)  12  1  2

(1) Return this exam copy with your exam booklet. (2) Write your solutions in your exam booklet. (3) Show your work. (4) There are six questions on this exam. (5) If you use a calculator it must be your own. (6) Round decimal answers to four decimal places. (7) You are expected to abide by the University’s rules concerning academic honesty.

1. (20 points) The population \( P(t) \), where \( t \) is years, of a city is growing exponentially at the rate of 18% per year. Given that \( P(4) = 2,000 \) find (a) \( P(t) \), (b) \( P(5) \), and (c) the continuous growth rate.

2. (15 points) Let \( y = f(x) = 4x^2 + 5x \). Starting with the difference quotient, use algebra to calculate \( f'(x) \).

3. (18 points) Sketch the graph of a function \( y = f(x) \) which has all of the following properties:
   
   (a) \( \lim_{x \to 0} f(x) = -\infty \), \( \lim_{x \to -3} f(x) = \infty \), \( \lim_{x \to 3^+} f(x) = -\infty \), \( \lim_{x \to -\infty} f(x) = 0 \);
   
   (b) \( f'(x) > 0 \) on the intervals \((-\infty, -2), (0, 3)\), and \((3, \infty)\);
   
   (c) \( f'(x) < 0 \) on the interval \((-2, 0)\);
   
   (d) \( f''(x) > 0 \) on the intervals \((-\infty, -3)\) and \((1, 3)\);
   
   (e) \( f''(x) < 0 \) on the interval \((-3, 0), (0, 1), \) and \((3, \infty)\).

   **You must label the numbers \(-3, -2, -1, 0, 1, 3\) on your \( x \)-axis.**

4. (17 points) Let \( f(x) = 5x + \frac{4}{x + 1} \). Then \( f'(x) = 5 - \frac{4}{(x + 1)^2} \).

   (a) Find an equation to the line tangent to the graph of \( y = f(x) \) at \( x = 9 \).
   
   (b) Assume that \( f(x) \) is the total cost of producing \( x \) items. Using the derivative, approximate the cost of producing the 11th item.

5. (15 points) Let \( f(x) = \begin{cases} x^3 - 4x^2 + 17 & : x \leq 2 \\ 3x + 5 & : x > 2 \end{cases} \). Compute (a) \( \lim_{x \to 2^-} f(x) \), (b) \( \lim_{x \to 2^+} f(x) \), and (c) determine whether or not \( y = f(x) \) is continuous at \( x = 2 \) from your limit calculations.

6. (15 points) A particle moving along a straight line has position \( s(t) = 2t^3 - 3t^2 - 12t + 1 \) at time \( t \), where \( t \geq 0 \). Given that \( s'(t) = 6t^2 - 6t - 12 \) and \( s''(t) = 12t - 6 \), find (a) when the particle is moving to the right, (b) to the left, and (c) when its velocity is increasing.