1. Consider the following statements:

(a) (not P) or (not Q);
(b) not (P or Q);
(c) not (P and Q);
(d) (not P) and Q.

Construct a truth table for each and determine which are logically equivalent.

2. Consider the following statements:

(a) P or (not Q);
(b) (not P) and Q.

Determine, by constructing a truth table or other means:

(a) if these statements are logically equivalent;
(b) if these statements are negations of each other;
(c) if one implies the other.

3. Consider the following statements:

(a) (P implies Q) implies R;
(b) P implies (Q implies R).
Determine, by constructing a truth table or other means:

(a) if these statements are logically equivalent;

(b) if one implies the other.

4. Consider the following universal statement: If $x$ is a real number and $x \geq 0$ then $x^2 > x$.

(a) Determine whether or not this universal statement is true.

(b) Determine whether or not the converse of this universal statement is true.

Construct tables similar to Table 2.1.2 of the text.

5. Let $a \in \mathbb{R}$, let $P$ be the statement “$a > 4$”, and let $Q$ be the statement “$a^2 - 3a - 4 \geq 0$”. Which of the following are true? In each case supply a proof or counterexample. For proofs you may assume Axiom 3.1.2 on page 24 of the text.

(a) $P$ implies $Q$.

(b) $Q$ only if $P$.

(c) $P$ is necessary for $Q$.

(d) $P$ if and only if $Q$.

(e) $P$ is sufficient for $Q$. 