1. **20 points total** Part (b) is a direct consequence of part (a).

(a) Suppose that $C \subseteq A, B$. Let $x \in C$. Then $x \in A$ since $C \subseteq A$ and $x \in B$ since $C \subseteq B$. (**2 points**) Since $x \in A$ and $x \in B$ we conclude $x \in A \cap B$. (**2 points**) We have shown that $x \in C$ implies $x \in A \cap B$. (**2 points**) Therefore $C \subseteq A \cap B$. (**2 points**)

(b) Suppose that C is a set, $n \geq 1$ and $A_1, \ldots, A_n$ are sets such that $C \subseteq A_1, \ldots, A_n$. Then $C \subseteq A_1 \cap \cdots \cap A_n$. We prove this assertion by induction on n.

Suppose that $n = 1$. Then $A_1 \cap \cdots \cap A_n = A_1$. Since $C \subseteq A_1$ by assumption, $C \subseteq A_1 \cap \cdots \cap A_n$. Thus the assertion is true for $n = 1$. (**2 points**)

Suppose that $n \geq 1$ and the assertion holds for n (induction hypothesis). Let $A_1, \ldots, A_{n+1}$ be sets such that $C \subseteq A_1, \ldots, A_{n+1}$. Then $C \subseteq A_1, \ldots, A_n$ and therefore $C \subseteq A_1 \cap \cdots \cap A_n$ by the induction hypothesis. (**4 points**) Since $C \subseteq A_{n+1}$ by assumption, by part (a)

$$C \subseteq (A_1 \cap \cdots \cap A_n) \cap A_{n+1} = A_1 \cap \cdots \cap A_{n+1}. \quad (\mathbf{2\ points})$$

We have shown that if the assertion holds for $n \geq 1$ then it holds for $n + 1$. (**2 points**) Therefore the assertion holds for all $n \geq 1$. (**2 points**)

2. **20 points total** In tabulated form:

(a)	$P(\emptyset)$	$=$	$\{\emptyset\}$	(**5 points**)
(b)	$P(\{7\})$	$=$	$\{\emptyset, \{7\}\}$	(**5 points**)
(c)	$P(\{\emptyset\})$	$=$	$\{\emptyset, \{\emptyset\}\}$	(**5 points**)
(d)	$P(\{6, 9\})$	$=$	$\{\emptyset, \{6\}, \{9\}, \{6, 9\}\}$	(**5 points**)

3. **20 points total**

$x \in A$	$x \in B$	$x \in C$	$x \in A \cup B$	$x \in (A \cup B) \cup C$	$x \in B \cup C$	$x \in A \cup (B \cup C)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

(**10 points**)

Since the columns under $x \in (A \cup B) \cup C$ and $x \in A \cup (B \cup C)$ are identical, $x \in (A \cup B) \cup C$ implies $x \in A \cup (B \cup C)$ (**4 points**) and $x \in A \cup (B \cup C)$ implies $x \in (A \cup B) \cup C$ (**4 points**). Therefore $A \cup (B \cup C) = A \cup (B \cup C)$ by definition of equality of sets. (**2 points**)

4. **20 points total** The completed table is

$x \in A$	$x \in B$	$x \in A \cap B$	$x \in (A \cap B)^c$	$x \in A^c$	$x \in B^c$	$x \in A^c \cup B^c$	
T	T	T	F	F	F	F	
T	F	F	T	F	T	T	(**10 points**)
F	T	F	T	T	F	T	
F	F	F	T	T	T	T.	

Since the columns under $x \in (A \cap B)^c$ and $x \in A^c \cup B^c$ are identical, $x \in (A \cap B)^c$ implies $x \in A^c \cup B^c$ (**4 points**) and $x \in A^c \cup B^c$ implies $x \in (A \cap B)^c$ (**4 points**). Therefore $(A \cap B)^c = A^c \cup B^c$ by definition of equality of sets. (**2 points**)

5. **20 points total** We are assuming that

$$(A^c)^c = A^{c\,c} = A \tag{1}$$

for all subsets $A \subseteq U$ and that

$$(A \cap B)^c = A^c \cup B^c \tag{2}$$

for all subsets $A, B \subseteq U$.

Let $A, B \subseteq U$. Then applying (2) to A^c and B^c, and then applying (1) gives

$$\begin{aligned} (A \cup B)^c &= ((A^c)^c \cup (B^c)^c)^c && (\textbf{7 points}) \\ &= ((A^c \cap B^c)^c)^c && (\textbf{7 points}) \\ &= A^c \cap B^c && (\textbf{6 points}). \end{aligned}$$