Let $A$ be a subset of the real numbers $\mathbb{R}$. A \textit{maximum for $A$} is a number $a \in A$ such that $a' \leq a$ for all $a' \in A$. A \textit{minimum for $A$} is a number $a \in A$ such that $a \leq a'$ for all $a' \in A$. We set $\neg A = \{ -a \mid a \in A \}$.

1. Suppose that $A$ is a subset of $\mathbb{R}$.
   
   (a) Show that $A$ has at most one maximum.

   (b) Let $a \in A$. Show that $a$ is a minimum for $A$ if and only if $-a$ is a maximum for $\neg A$.

   (c) Use parts (a) and (b) to show that $A$ has at most one minimum.

2. Suppose that $A$, $B$ are subsets of $\mathbb{R}$.

   (a) Suppose that $A \cup B$ has a maximum. Show that either $A$ has a maximum or $B$ has a maximum.

   (b) Suppose that $A$ and $B$ each has a maximum. Show that $A \cup B$ has a maximum.

3. Let $U$ be a universal set and $A, B \subseteq U$.

   (a) Show that $\chi_{A \cap B} = \chi_A \chi_B$.

   (b) Show that $\chi_{A^c} = 1 - \chi_A$.

   (c) Use parts (a) and (b) to show that $\chi_{A - B} = \chi_A(1 - \chi_B)$.

4. We continue with Problem 3.
(a) Show that \( \chi_{A \cup B} = \chi_A + \chi_B - \chi_{A \cap B} \) (\( = \chi_A + \chi_B - \chi_A \chi_B \) by part (a) of Problem 3).

(b) Using the fact that \( A = B \) if and only if \( \chi_A = \chi_B \), use parts (a) and (b) of Problem 3 and part (a) to prove De Morgan’s Law \( (A \cup B)^c = A^c \cap B^c \).

5. Find the greatest common divisor of

(a) 22 and 234;

(b) 39 and 385;

(c) 16 and 120.