Math 215, Fall 05 Homework #3

Due Friday, 09/16/05 at the beginning of class

You may assume the propositions and theorems of the text. You may assume that the product of two positive real numbers, and the product of two negative real numbers, is positive. You may also assume that the product of a positive real number and a negative real number is negative.

1. Let n be an integer. Consider the assertion: If $0 < n < 3$ then $n^3 < -n^2 + 12n$.

   a) Prove the assertion by cases.

   b) Prove the assertion by "working backwards".

   c) Show that $-4 < n < 3$ and $n \neq 0$ is equivalent to $n^3 < -n^2 + 12n$.

2. Let a be a real number. Consider the assertion: $a^2 \geq 9a$ implies $a \leq 0$ or $a \geq 9$.

   a) Prove the assertion by contradiction.

   b) Prove the assertion directly.

3. Let a be a real number. Consider the assertion: $a^2 - 12a + 32 < 0$ implies $4 \leq a \leq 8$.

   a) Prove the assertion by contradiction.

   b) Prove the assertion directly.

   c) Is the converse of the assertion true? Give a proof or counterexample.

4. Let S be a non-empty set with a given multiplication which we denote by "dot". Thus if $a, b \in S$ then their "product" is denoted by $a \cdot b \in S$. Define a new multiplication $\cdot^{op}$ in terms the given one by

$$a \cdot^{op} b = b \cdot a$$

for all $a, b \in S$.

a) Show that the given multiplication is associative if and only if the new one is.

b) Show that the given multiplication is commutative if and only if the new one is.

c) Suppose that there are elements $e, e' \in S$ such that $a \cdot e = a$ and $e' \cdot a = a$ for all $a \in S$. Show that $e = e'$; hence $a \cdot e = a = e \cdot a$ for all $a \in S$.