Math 215, Fall 05 Homework #9

Due Friday, 11/04/05 at the beginning of class. Problem 3a), c) revised 11/03/05

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1. The purpose of this exercise is to construct a proof of Corollary 10.2.2.

   a) Let $n \geq 1$ and $X_1, \ldots, X_n, A$ be sets such that $X_1 \cap A = \cdots = X_n \cap A = \emptyset$. Show, by induction on n, that $(X_1 \cup \cdots \cup X_n) \cap A = \emptyset$. The union $X_1 \cup \cdots \cup X_n$ is defined inductively by

   $$X_1 \cup \cdots \cup X_n = \begin{cases} X_1 & : \quad n = 1 \\ (X_1 \cup \cdots \cup X_{n-1}) \cup X_n & : \quad n > 1 \end{cases} .$$

   b) Use Proposition 10.2.1 and part a) to establish Corollary 10.2.2 by induction on n.

2. Let X and Y be finite sets.

   a) Suppose that $X \subseteq Y$ and $|X| = |Y|$. Use Theorem 10.2.1 to establish $X = Y$. [Hint: $Y = X \cup (Y - X)$.]

   b) Suppose $|X \cup Y| = |X \cap Y|$. Use part a) to show that $X = Y$.

3. This exercise is about the inclusion-exclusion principle.

   a) Let X and Y be finite sets and suppose that $|X| = 11$, $|Y| = 6$, and $|X \cap Y| = 4$. Find $|X \cup Y|$.

   b) Suppose that U is a finite universal set. If $|U| = 21$, $|X \cup Y| = 11$, $|X| = 4$, and $|Y| = 10$, find $|X^c \cup Y^c|$.

   c) Each tile in a collection of 19 is a square or a triangle and is also red or blue. Suppose that 12 of the 19 tiles are squares, 11 are red, and 4 are blue squares. Using the inclusion-exclusion principle, determine:

(1) the number of tile which are square or blue;

(2) the number of tiles which are triangles and red;

(3) the number of tiles which are red or squares.

[Hint: Let U be the (universal) set of all the 19 tiles and let S, T, R, and B denote the subset of tiles which is a square (respectively a triangle, red, and blue). Then $T = S^c$ and $R = B^c$. Part (2) comes down to computing $|T \cap R|$ and part (3) to determining $|B^c \cup T^c|$.]

4. Consider the following types of currency: \$1, \$5, \$10, \$20, \$50, and \$100. Suppose that X is a set of people. Show that if $|X| \geq 8$ then some pair of people in X have no currency of these types or the two have exactly the same number of these types of currency. Use Theorem 11.1.2 (the pigeonhole principle) to construct a proof along the lines of Example 11.1.3(b).