1. (12 pts.) Find the unique solution to \( y'' - y' - 6y = 0 \), where \( y(0) = 2 \) and \( y'(0) = -3 \).

**Solution:** The auxiliary equation is \( r^2 - r - 6 = 0 \). Since \( r^2 - r - 6 = (r - 3)(r + 2) \) the auxiliary equation has two distinct real roots \( r = 3, -2 \) (3 points). Therefore the general solution is

\[
y = c_1 e^{3x} + c_2 e^{-2x}. \quad (4 \text{ points})
\]

Since

\[
y' = 3c_1 e^{3x} - 2c_2 e^{-2x}
\]

we need to solve the system

\[
\begin{align*}
2 & = y(0) = c_1 + c_2 \\
-3 & = y'(0) = 3c_1 - 2c_2
\end{align*}
\]

which has solution

\[
c_1 = \frac{1}{5} \quad \text{and} \quad c_2 = \frac{9}{5}.
\]

Therefore

\[
y = \frac{1}{5} e^{3x} + \frac{1}{5} e^{-2x} \quad (4 \text{ points}).
\]

2. (8 pts.) Find the general solution to \( y'' + y' + 5y = 0 \).

**Solution:** The auxiliary equation is \( r^2 + r + 5 = 0 \) which has complex roots \( r = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i \) (4 points). Therefore the general solution is

\[
y = c_1 e^{-\frac{1}{2}x} \cos(\frac{\sqrt{19}}{2}x) + c_2 e^{-\frac{1}{2}x} \sin(\frac{\sqrt{19}}{2}x). \quad (4 \text{ points}).
\]
Name (print) ________________________________________________

Discussion (circle day, time)  Tu  Th  10  12

(1) Show your work for full credit.  (2) Give exact answers whenever possible; otherwise give answers accurate to two decimal places.  (3) You are expected to abide by the University’s rules concerning academic honesty.

1. (12 pts.) Find the unique solution to \( y'' - y' - 20y = 0 \), where \( y(0) = -1 \) and \( y'(0) = 2 \).

Solution: The auxiliary equation is \( r^2 - r - 20 = 0 \). Since \( r^2 - r - 20 = (r - 5)(r + 4) \) the auxiliary equation has two distinct real roots \( r = 5, -4 \) (4 points). Therefore the general solution is

\[
y = c_1 e^{5x} + c_2 e^{-4x}. \quad (4 \text{ points})
\]

Since

\[
y' = 5c_1 e^{5x} - 4c_2 e^{-2x}
\]

we need to solve the system

\[
\begin{align*}
-1 &= y(0) = c_1 + c_2 \\
2 &= y'(0) = 5c_1 - 4c_2
\end{align*}
\]

which has solution

\[
c_1 = -\frac{2}{9} \quad \text{and} \quad c_2 = -\frac{7}{9}.
\]

Therefore

\[
y = -\frac{2}{9} e^{5x} - \frac{7}{9} e^{-4x} \quad (4 \text{ points}).
\]

2. (8 pts.) Find the general solution to \( y'' + 3y' + 4y = 0 \).

Solution: The auxiliary equation is \( r^2 + 3r + 4 = 0 \) which has complex roots \( r = -\frac{3}{2} \pm \frac{\sqrt{7}i}{2} \) (4 points). Therefore the general solution is

\[
y = c_1 e^{-\frac{3}{2}x} \cos\left(\frac{\sqrt{7}}{2} x\right) + c_2 e^{-\frac{3}{2}x} \sin\left(\frac{\sqrt{7}}{2} x\right). \quad (4 \text{ points}).
\]