

# Revised Syllabus for the Final and Study Guide.

11/28/04 Radford

The final examination will be on Chapters 1–16, as indicated below. You have a lot of written material to use in your study at this point; see our course home page.

## Chapter 0 3–23

- *Basic Concepts:* Well-ordering, induction, divides, divisor, greatest common divisor, Division Algorithm for integers, Euclidean Algorithm, factor, multiple, least common multiple, equivalence relations and classes, functions, one-one, onto, inverse function.
- *Particular Results:*
- *Problems of Note:*
- *Omit:* Examples 5, 6, 8, 10.
- *Comments:* The material of Chapter 0 is considered prerequisite.

## Chapter 1 31–37.

- *Basic Concepts:* Cayley table.
- *Particular Results:*
- *Problems of Note:* Construction of Cayley tables for groups.
- *Omit:*
- *Comments:*

## Chapter 2 42–52

- *Basic Concepts:* Binary operation, closure, group.
- *Particular Results:*
- *Problems of Note:*

- *Omit*: Examples 14, 20.
- *Comments*: The examples are particularly good. Be sure to understand the table on page 52. Understand how to show closure.

### Chapter 3 58–67

- *Basic Concepts*: Order of group, order of element, subgroup, multiplicative and additive notations.
- *Particular Results*: Tests for subgroups, when a finite subset of a group is a subgroup.
- *Problems of Note*: Know how to determine whether or not a subset of a group is a subgroup.
- *Omit*: Example 11.
- *Comments*:

### Chapter 4 73–82

- *Basic Concepts*:
- *Particular Results*:
- *Problems of Note*:
- *Omit*: Nothing.
- *Comments*: See the course notes on cyclic groups also. This topic is absolutely fundamental.

### Chapter 5 93–103

- *Basic Concepts*: Permutation of a set, the permutation group of a set, symmetric group, decomposition into product of disjoint cycles,  $n$ -cycle, odd permutation, even permutation, the alternating group.
- *Particular Results*: Every permutation of a finite set is a product of disjoint cycles, is a product of 2-cycles.
- *Problems of Note*: Find orders of permutations, write permutations as products of disjoint cycles, as products of 2-cycles.

- *Omit:*
- *Comments:*

### Chapter 6 118–125

- *Basic Concepts:* Isomorphism of groups.
- *Particular Results:* Cayley's Theorem.
- *Problems of Note:* Know how to show that a function  $f : G \longrightarrow G'$  of groups is a group isomorphism, is not a group isomorphism.
- *Omit:* Example 8.
- *Comments:*

### Chapter 7 134–138

- *Basic Concepts:* Left coset, right coset, of a subgroup.
- *Particular Results:* Basic properties of left cosets, Lagrange's Theorem and consequences.
- *Problems of Note:* Calculate left, right, cosets.
- *Omit:*
- *Comments:* This material is extremely important.

### Chapter 8 150–156

- *Basic Concepts:* External direct product, order of element in direct product.
- *Particular Results:* When a direct product is cyclic.
- *Problems of Note:* Find cyclic subgroups of a direct product of given order.
- *Omit:* Theorem 8.3 to rest of chapter.
- *Comments:*

### Chapter 9 172–179

- *Basic Concepts:* Normal subgroup, factor group.

- *Particular Results:*
- *Problems of Note:* Determine whether or not a subgroup is normal.
- *Omit:* Examples 4, 9, 10.
- *Comments:* Cosets may be represented in different ways.

### Chapter 10 194–205

- *Basic Concepts:* Group homomorphism, kernel of a group homomorphism,
- *Particular Results:* Kernels are normal subgroups and normal subgroups are normal. The First Isomorphism Theorem for groups.
- *Problems of Note:* Know how to show that a function  $f : G \rightarrow G'$  of groups is a group homomorphism, is not a group homomorphism.
- *Omit:* Examples 11, 12, 15, 16.
- *Comments:* Understand the concept of *well-defined*. The problem of determining all group homomorphisms  $f : G \rightarrow G'$  of finite-cyclic groups is an excellent application of most of the fundamental ideas concerning cyclic groups.

### Chapter 12 229–234

- *Basic Concepts:* Ring, ring with unity, units of a ring with unity, the group of units  $R^*$  of a ring  $R$  with unity, subring, direct sum of rings.
- *Particular Results:*
- *Problems of Note:* Determine whether or not a subset of a ring  $R$  is a subring.
- *Omit:*
- *Comments:* Rings are additive abelian groups and subrings are additive subgroups. Thus the theory of rings builds on the theory of abelian groups.

### Chapter 13 240–246

- *Basic Concepts:* Zero divisor, integral domain, field, characteristic of a ring with unity.
- *Particular Results:* Fields are integral domains. Finite integral domains are fields.  $\mathbf{Z}_n$  is an integral domain (hence a field) if and only if  $n$  is prime.
- *Problems of Note:* Determine whether or not a commutative ring with unity is an integral domain, is a field.
- *Omit:*
- *Comments:*

#### Chapter 14 253–260

- *Basic Concepts:* Ideal, factor ring, prime ideal, maximal ideal.
- *Particular Results:*  $R$  a commutative ring with unity,  $I$  an ideal of  $R$ . Then  $R/I$  is an integral domain (resp. field) if and only if  $I$  is a prime (resp. maximal) ideal of  $R$ .
- *Problems of Note:* Determine whether or not a subset  $I$  of a ring  $R$  is an ideal of  $R$ . Ideals of  $\mathbf{Z}$ .
- *Omit:* Examples 10, 11.
- *Comments:* Every ideal of a ring is a subring and hence an additive subgroup.

#### Chapter 15 270–277

- *Basic Concepts:* One-one, onto, ring homomorphism, kernel.
- *Particular Results:* Basic properties of ring homomorphisms, every kernel is an ideal, and every ideal is a kernel, first isomorphism theorem for rings. Every integral domain is a subring of a field (the field of quotients).
- *Problems of Note:* Know how to show that a function  $f : R \rightarrow R'$  of rings is a ring homomorphism, is not a ring homomorphism.
- *Omit:* Examples 8, 9.
- *Comments:* Every ring homomorphism is a homomorphism of additive groups. Thus the theory of ring homomorphisms builds on the theory of group homomorphisms.

## Chapter 16 283–290

- *Basic Concepts:* Ring of polynomials  $R[x]$  over a commutative ring  $R$ , degree of polynomial, monic polynomial, leading coefficient, constant polynomial.
- *Particular Results:*  $R$  a division ring implies  $R[x]$  is also. The Division Algorithm for  $F[x]$ . Remainder and Factor Theorems. The structure of ideals of  $F[x]$ .
- *Problems of Note:* Consequences of an analog of the Euclidean Algorithm for integers. Finding quotients and remainders in  $F[x]$  for various fields  $F$ .
- *Omit:*
- *Comments:* The ideal structure of  $F[x]$  and the subgroup structure of  $\mathbf{Z}$  are similar by virtue of the division algorithms.