Homework # 3 (week of 09/06–09/10)

Due Friday, 09/10/04 in class

1. Let $G = \text{GL}(2, \mathbb{R})$ be the group of $2 \times 2$ invertible matrices with real coefficients under matrix multiplication.

   a) Show that $G$ is not abelian by finding specific $g, g' \in G$ such that $gg' \neq g'g$.

   b) Find all $g \in G$ such that $gx = xg$ for all $x \in G$.

   c) Determine whether or not $H = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in \mathbb{R}, ac \neq 0 \right\}$ is a subgroup of $G$.

2. Let $G = \text{U}(9)$.

   a) Write out the Cayley table for $G$.

   b) Find a $g \in G$ such that $G = \langle g \rangle$.

3. Prove that $G = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}$ is a group under matrix multiplication. (You may assume that matrix multiplication is associative.)