1. Suppose that $G$ is a finite group with subgroups $H$, $K$. Prove that if $|H|$ and $|K|$ are relatively prime then $H \cap K = \langle e \rangle$.

2. Let $f : G \rightarrow G$ be a group homomorphism, let $S$ be a non-empty subset of $G$ and suppose that $f(S) \subseteq \langle S \rangle$. Show that $f(\langle S \rangle) \subseteq f(\langle S \rangle)$.

3. The group $\text{GL}_2(\mathbb{R})$ of $2 \times 2$ invertible matrices with coefficients in the real numbers $\mathbb{R}$ acts on $A = \mathbb{R}^2$ by $g \cdot v = gv$ for $g \in G$ and $v \in A$. Let $G = \left\{ \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix} \mid b \in \mathbb{Q} \right\}$.
   a) Show that $G$ is a subgroup of $\text{GL}_2(\mathbb{R})$.
   b) Show that the $G$-orbits of $A$ have the form $L_y = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} \right\}$, where $y \in \mathbb{R}$, or $U_x = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \right\}$, where $x \in \mathbb{R} \setminus 0$.

4. Let $G = S_6$, $\sigma = (1 \ 3 \ 2 \ 5)(4 \ 1 \ 6) \in G$ and $H = \langle \sigma \rangle$.
   a) List all of the elements of $H$ as products of disjoint cycles.
   b) Show that $H$ is not a normal subgroup of $G$. [Hint: Consider $\tau \sigma \tau^{-1}$, where $\tau$ has order 3.]
   c) Let $\tau = (1 \ 5)(2 \ 6)(3 \ 4)$. Show that $\tau \sigma \tau^{-1} \in H$. [Thus $\tau \in N_G(H)$ by Problem 2.]
   d) Show that $12$ divides $|N_G(H)|$.

5. Let $G$ be a group of order 3·5·19.
   a) Show that $G$ has a unique subgroup of order 5 and a unique subgroup of order 19.
   b) Show that $G$ has a subgroup of index 3. [Hint: You may use the fact that if $H, K \leq G$ and $H \leq G$ then $HK \leq G$.]