Math 517 **Spring 2007** **Radford**

Written Homework # 4

Due at the beginning of class 04/13/07

If F and K are fields then $F \subseteq K$ means that F is a subfield of K. Also $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{C}$ denote the fields of rational, real, and complex numbers respectively. You must justify your answers.

1. Find:

(1) The degree of $\sqrt[10]{34}$ over $\mathbf{Q}$;

(2) $\mathrm{m}_{\mathbf{Q}[\sqrt[3]{21}],\,\sqrt[10]{34}}(x)$;

(3) $\mathrm{m}_{\mathbf{Q}[\sqrt[10]{34}],\,\sqrt[3]{21}}(x)$.

2. Let $a = \sqrt[3]{n} \in \mathbf{R}$, where $n \geq 2$ and is an integer such that p divides n but p^2 does not divide n for some prime p. Let $K = \mathbf{Q}[a]$.

(1) Determine $[K : \mathbf{Q}]$ and find a basis for K as a vector space over $\mathbf{Q}$.

(2) Let $b = r + sa$, where $r, s \in \mathbf{Q}$ and $s \neq 0$. Show $b \notin \mathbf{Q}$ and find $\mathrm{m}_{\mathbf{Q},b}(x)$.

3. Let $a = \sqrt{1 + \sqrt{2}} \in \mathbf{R}$.

(1) Show that $a \notin \mathbf{Q}[\sqrt{2}]$.

(2) Find $\mathrm{m}_{\mathbf{Q},a}(x)$ and find $\mathrm{m}_{\mathbf{Q}[\sqrt{2}],a}(x)$.

(3) Write $f(x) = \mathrm{m}_{\mathbf{Q},a}(x)$ as a product of linear factors over $\mathbf{C}$, determine a splitting field $K \subseteq \mathbf{C}$ for $f(x)$ over $\mathbf{Q}$, and determine $[K : \mathbf{Q}]$.

4. Let F and K be fields and $F \subseteq K$.

(1) Show that all $a \in K \backslash K_{alg}$ are transcendental over K_{alg}. (Thus a is transcendental over F.)

(2) Suppose that $a \in K$ is transcendental over F. For $n \geq 1$ show that a^n is transcendental over F and that $[F(a) : F(a^n)] = n$. (Thus $F(a)$ is an algebraic extension of $F(a^n)$ of degree n.)