If $F$ and $K$ are fields then $F \subseteq K$ means that $F$ is a subfield of $K$. Also $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ denote the fields of rational, real, and complex numbers respectively.

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a homomorphism of rings with unity. Show that $f = \text{Id}_\mathbb{R}$ (thus $\text{Aut}(\mathbb{R})$ is trivial). [Hint: Show that $f$ is order preserving. You can assume the following facts from Analysis: $a \leq b$ if and only if $b - a$ is a square and there is a rational number between two different real numbers.]

2. Let $F \subseteq K$, where $F$ is finite, $|F| = p^m$, and $[K : F] = n$. Show that the product of all monic irreducible polynomials $p(x) \in F[x]$ whose degree divides $n$ is $x^{\text{ord}_K} - x$.

3. Let $K = \mathbb{Q}[a, b, \omega] \subseteq \mathbb{C}$, where $a, b \in \mathbb{R}$ satisfy $a^3 = 5$, $b^5 = 6$, and $\omega \in \mathbb{C}$ is a primitive $3^{rd}$ root of unity.
   (1) Find $[K : \mathbb{Q}]$.
   (2) Determine the group $\text{Aut}(K)$.
   (3) Is $K$ a Galois extension of $\mathbb{Q}$? Of $\mathbb{Q}[b]$?
   (4) What is the smallest closed subfield of $K$?

4. Find the Galois group of $f(x) = x^4 - 30$ over
   (1) $\mathbb{Q}$, and
   (2) over $\mathbb{Q}[i]$ by determining generators and relations.