Math 517 **Spring 2007** **Radford**

Written Homework # 5

Due at the beginning of class 05/04/07

If F and K are fields then $F \subseteq K$ means that F is a subfield of K. Also $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{C}$ denote the fields of rational, real, and complex numbers respectively.

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1. Let $f : \mathbf{R} \longrightarrow \mathbf{R}$ be a homomorphism of rings with unity. Show that $f = \mathrm{Id}_{\mathbf{R}}$ (thus $\mathrm{Aut}(\mathbf{R})$ is trivial). [Hint: Show that f is order preserving. You can assume the following facts from Analysis: $a \leq b$ if and only if $b-a$ is a square and there is a rational number between two different real numbers.]

2. Let $F \subseteq K$, where F is finite, $|F| = p^m$, and $[K : F] = n$. Show that the product of all monic irreducible polynomials $p(x) \in F[x]$ whose degree divides n is $x^{p^{mn}} - x$.

3. Let $K = \mathbf{Q}[a, b, \omega] \subseteq \mathbf{C}$, where $a, b \in \mathbf{R}$ satisfy $a^3 = 5$, $b^5 = 6$, and $\omega \in \mathbf{C}$ is a primitive 3^{rd} root of unity.

(1) Find $[K : \mathbf{Q}]$.

(2) Determine the group $\mathrm{Aut}(K)$.

(3) Is K a Galois extension of $\mathbf{Q}$? Of $\mathbf{Q}[b]$?

(4) What is the smallest closed subfield of K?

4. Find the Galois group of $f(x) = x^4 - 30$ over

(1) $\mathbf{Q}$, and

(2) over $\mathbf{Q}[\imath]$

by determining generators and relations.