EXERCISE III-STAT 571-Fall 2003

I. Joe and Pat have $2000 each in their checking accounts. Joe works for a gambling house and Pat works for a stock trader. They are discussing what to do with their money in the bank. Joe could just do nothing or encourage Pat to invest their bank accounts for investment. If he does, Pat has the option to either invest the money with her stock trader or encourage Joe to play a game of matching penny at the gambling house. She knows that by stock trading jointly they will only break even and yet she has to pay $1000 to Joe on top of his investment. If she decides to gamble, he has no choice than to gamble with her. Each one will toss an ordinary coin and when the outcomes match, Pat closes her checking account with a single check for $2000 to Joe. Otherwise Joe closes his checking account with a single check to Pat. The rules are common knowledge for both Pat and Joe. Form an extensive game and find a subgame perfect Nash equilibrium strategy for the game. II. JOE and PAT are neighbors and JOE can see from his window whether PAT is watching morning TV. They go to church service on Sundays. On Saturday night they both heard that it is 50% chance for rain or shine on Sunday. Each one is lazy to carry an umbrella. If it rains and they go without umbrella, each one may have to catch a taxi while coming home, costing $5 each. In case one has an umbrella and it rains, that person can catch a bus and come home costing $2. If it shines and the person has an umbrella, the person will spend $1 on an ice cream. If there is sunshine and if the person carries no umbrella, the freedom is worth $1 to the person. Form an extensive game under two assumptions
   i. PAT did not watch the weather report before leaving home.
   ii. JOE saw PAT watching the weather report.
iii. Find a Nash equilibrium strategy pair if any by dominance solvability for the two games. III. Find the behavior strategy for player I who uses the mixed strategy that chooses Lps with probability .7 and Mqr with probability .3 in the game myopic2.bmp (this was supplied in class last time). IV. Let \( K_i(\sigma_1, \sigma_2, \ldots, \sigma_n), \quad i = 1, \ldots, n \) be the strategic form payoffs when players 1, 2, \ldots, n choose secretly pure strategies \( \sigma_1 \in \Sigma_1, \ldots, \sigma_n \in \Sigma_n \) respectively. We will assume \( \Sigma_1, \Sigma_2, \ldots, \Sigma_n \) finite. If \( x_1^*, x_2^*, \ldots, x_n^* \) is a Nash equilibrium point in mixed strategies, show that

\[
K_i(\sigma, x_2^*, \ldots, x_n^*) = K_i(x_1^*, x_2^*, \ldots, x_n^*)
\]

if \( x_1^* \) selects pure strategy \( \sigma \) with positive probability. V. Find a Nash equilibrium point to the bimatrix game:

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[7, 43, 95, 87, 3]
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