Note Title

Math 160, Spring 2010 - W11 Fri Lec Notes

4/4/2010

Probability Review Problems:

P1: Given 10 distinct books, 10 are red, how many ways can the books be arranged on a shelf if the red books must stay together?

<u>General guideline for counting problems:</u>

Find a <u>Process</u> that has 3 properties

(a) If you follow the process one time the result will be one possible arrangement.

(b) If you follow the process in all possible ways, you will get all possible outcomes

(c) You can count the number of ways to complete the process with the counting techniques learned so far.

Here is a process that will work for this problem:

Process: 1st tie the red books together and arrange the resulting 8 objects (7 books that are not

red and one block of red books.
There 8! ways to complete this step.

2nd Untie the red books and arrange them in space that the occupy.
There are 3! ways to complete this step.

Answer: $N = 8! \cdot 3!$ This is not the only process that will work.

Here are some important results that may be useful in the next problem.

General Addition Rule (always correct)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Product Rule (always correct)

Version 1:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Version 2:

$$P(A \cap B) = P(B) \cdot P(A|B)$$

Both versions are a result of the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The multiplication and addition rules Reduce to special forms under certain conditions:

If A, B are mutually Exclusive
Then $A \cap B = \emptyset$ and $P(A \cap B) = 0$.

The Addition Rule reduces to:

$$P(A \cup B) = P(A) + P(B)$$

If A, B are Independent then
Product Rule Reduces to:

$$P(A \cap B) = P(A) \cdot P(B)$$

Adition Rule Reduces to:

$$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

these variations can be useful when using Trees, Tables, Venn Diagrams or formulas to solve problems.

Example: Given the probability of at least one of A, B is 3/5, A and B are independent and P(B) = 1/5. Find the P(A) = ?

You should 1st decide on a method to organize the problem: tree, table, Venn Diagram or just use Rules and definitions. As a general rule if conditional probabilities are given, then a tree might be a good choice. If conditional probabilities are not given then a table or Venn Diagram might be a good choice.

First try: Use a table to organize the problem. Since A, B are independent then so are all of $A, B, \bar{A}, \bar{B}$ independent.

So:

$P(A \cap B) = P(A) \cdot P(B)$ $\quad$ $P(A \cap \overline{B}) = P(A) \cdot P(\overline{B})$

$P(\overline{A} \cap B) = P(\overline{A}) \cdot P(B)$ $\quad$ $P(\overline{A} \cap \overline{B}) = P(\overline{A}) \cdot P(\overline{B})$

ie. The Probability of each cell in the table is the product of the Marginal Probabilities.

Set up the table for problem:

	B	$\overline{B}$	
A	$\frac{1}{5}\cdot P(A)$	$\frac{4}{5}\cdot P(A)$	$P(A)$
$\overline{A}$	$\frac{1}{5}(1-P(A))$	$\frac{4}{5}(1-P(A))$	$P(\overline{A}) = 1 - P(A)$
	$\frac{1}{5}$	$\frac{4}{5}$	1

$+\ P(A \cup B) = \frac{3}{5}$ given. $\Longrightarrow$

$$P(A \cup B) = \frac{3}{5} = \frac{1}{5}(1 - P(A)) + \frac{1}{5} \cdot P(A) + \frac{4}{5} P(A)$$

solve for $P(A)$ and get $P(A) = \frac{1}{2}$.

Note: for this problem using table was probably not the fastest way to solve the problem but it worked.

Here is an alternate way to work the problem:

Given $P(A \cup B) = \frac{3}{5}$, $P(B) = \frac{1}{5}$

find $P(A) = ?$ use formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ independent

$$= P(A) + P(B) - P(A) \cdot P(B)$$

$$\frac{3}{5} = P(A) + \frac{1}{5} - P(A) \cdot \frac{1}{5}$$

Solve for $P(A)$ get $P(A) = \frac{1}{2}$.

Note: a Venn Diagram, or a tree can also be used to solve the problem. In this case using the addition rule was probably the easiest. It is not so important that you always use the easiest method. What is important is that get skillful at solving basic problems using different methods.

Example: Given A, B are independent,

$P(A) = \frac{1}{3}$, $P(\overline{B}) = \frac{1}{4}$.

Find $P(\overline{A \cup B}) = ?$

$P(\overline{A} \cap \overline{B}) = ?$

Are A, B mutually Exclusive?

It is easy to answer all questions by using formulas, definitions. However, a table is useful.

	B	$\bar{B}$	
A	$\frac{3}{4}\cdot\frac{1}{4}=\frac{3}{12}$	$\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$	$\frac{1}{3}$
$\bar{A}$	$\frac{3}{4}\cdot\frac{2}{3}=\frac{6}{12}$	$\frac{1}{4}\cdot\frac{2}{3}=\frac{2}{12}$	$2/3$
$\bar{A}\cup B$	$3/4$	$\frac{1}{4}$	1

A, B ind $\Rightarrow$ Mult. Margins to get prob of cells.

Note:

(a) $\sum$ cells $= 1$

(b) $\sum$ each margin adds to 1.

$$P(\overline{A} \cup B) = \frac{3}{12} + \frac{6}{12} + \frac{2}{12} = \frac{11}{12}$$

OR you can use the Addition Rule with the table information.

$$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cup B)$$

$$= \frac{2}{3} + \frac{3}{4} - \frac{6}{12}$$

$$= \frac{8}{12} + \frac{9}{12} - \frac{6}{12} = \frac{11}{12}.$$

$$P(\bar{A} \cap \bar{B}) = \frac{2}{12} = \frac{1}{6} \quad \text{from table}$$

Here is how you can solve the problem with formulas only:

Given $P(A) = \frac{1}{3}$, $P(\bar{B}) = \frac{1}{4}$, A and B independent.

Find $P(\bar{A} \cup B) = ?$

$$P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B) \quad \text{ind}$$

$$= P(\bar{A}) + P(B) - P(\bar{A}) \cdot P(B)$$

$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B) = 1 - P(\bar{B}) = 1 - \frac{1}{4} = 3/4$

$$P(\bar{A} \cup B) = \frac{2}{3} + \frac{3}{4} - \frac{2}{3} \cdot \frac{3}{4} = \frac{11}{12}$$

Note: You never need tables, trees Venn Diagrams etc. However, they can help if you are having trouble with a problem.

Last question: Are A, B mutually exclusive?

Answer: No because $P(A \cap B) = \frac{3}{12} \neq 0$