Bayes' Theorem:
The oil drilling problem from the last lecture is an example of a Bayes Theorem type of problem. The given conditional probabilities were of form: $P(+|\text{L})$ and $P(-|\text{L})$. However, the probabilities to be found
were of form \( P(0|+) \)

\[
\begin{align*}
\text{ie } P(0|+) &= \frac{P(0,+)\cdot P(+|0)}{P(+)} \\
&= \frac{P(0, P(+|0)}{P(0). P(+|0) + P(0). P(+|10)}
\end{align*}
\]

This is a special case of Bayes' formula. The probabilities were...
found from a tree and Bayes' formula was not explicitly used.

Two ways to get a positive test result.

When there many branches in
The 1st stage of the tree it is often convenient to use a Bayes' Formula.
Here is a simple example that demonstrates how to derive Bayes' Formula.
Assume a factory has three
machines that make parts: $m_1, m_2, m_3$

Each machine makes defective (D) parts and good parts (G). From past records these probabilities are known:

$P(m_1)$ probability (fraction) of parts made by machine 1.

$P(m_2)$ probability a part is made by $m_2$.

$P(m_3)$ fraction of parts made by $m_3$. 
It is also known what the probabilities are that each machine will produce a defective part:

\[ P(D \mid m_1), P(D \mid m_2), P(D \mid m_3) \]

A tree can be used to organize the information.
Three ways a defective part can be produced.

Note: \( P(m_1) + P(m_2) + P(m_3) = 1 \)
Now, at the end of the day a part is chosen at random and it is defective. What is the probability that it was made by machine 1? 

From the tree and the definition of conditional probability
\[
P(D|m_i) = \frac{P(m_i, n, d) - P(m_i, n, d)}{P(D)} = \frac{P(m_i, n, d)}{P(m_i, n, d) + P(m_2, n, d) + P(m_3, n, d)} \]

\[
= \frac{P(m_i) \cdot P(D|m_i)}{P(m_1) \cdot P(D|m_1) + P(m_2) \cdot P(D|m_2) + P(m_3) \cdot P(D|m_3)} \]

\[
= \frac{P(m_i) \cdot P(D|m_i)}{\sum_{i=1}^{3} P(m_i) \cdot P(D|m_i)}
\]
You can ask the same question for machines $i = 1, 2, 3$, giving a more general formula

Bayes' Formula

$$P(m_i | D) = \frac{P(m_i) \cdot P(D | m_i)}{\sum_{i=1}^{n} P(m_i) \cdot P(D | m_i)}$$
If you remember and understand this problem then you can easily reproduce Bayes' Formula when you need it. In addition, have a good sense for how to use it.

Here are two practice problems for probability. They are not Bayes' problems.
Example: Awards will be given to seven people whose names start with letters J, K, L, M, N, O and P. In how many ways can the awards be presented if the second award is presented to either K, J or N?

In counting problems where there are restrictions, it is usually best to follow a process that takes care
of the restrictions first. This usually simplifies the problem.

Process: Assign people to the awards one thru seven.

1st 2nd 3rd 4th 5th 6th 7th.

1st Assign a person to the 2nd award. There are 3 ways to do this.

2nd There are now 6 people to be assigned to the 6 remaining
awards. This can be done in 6! ways.

\[ N = 3 \cdot 6! \]

Example: Two year old Andy helped his mother unpack groceries. In the process he removed the labels from all of the cans. Five
Cans contained peaches, six cans contained vegetables, five contained soup, and three contained dog food. His mother plans to open three cans for dinner. What is the probability that she will open exactly one can of vegetables?

A good way to start is to draw a picture.
5 Peaches
6 Veggies
5 Soup
3 Dog Food

$N = 19$

$\Rightarrow$ Pick 3 at random.

Order does not matter.

In this case the problem can be simplified because the question only asks about getting veggies or not. Re-draw the box.
as \[ \begin{bmatrix} 6 \\ 13 \end{bmatrix} \rightarrow \text{Pick 3 at Random,} \]
\[ N = 19 \]

\[ P(\text{only one can of Veggies}) \]
\[ = P(1V \text{ and } 2\bar{V}) = \frac{n(1V \text{ and } 2\bar{V})}{n(\text{any 3 cans})} \]

\[ = \frac{n(1V) \times n(2\bar{V})}{n(\text{any 3 from 19})} \]
\[
\begin{align*}
&\left( \begin{array}{cc}
9 & 1 \\
3 & 1
\end{array} \right) \\
&\left( \begin{array}{c}
6 \\
1\end{array} \right) \cdot \left( \begin{array}{c}
13 \\
2
\end{array} \right) \\
&\left( \begin{array}{c}
1, 9 \\
3
\end{array} \right) \quad \text{(Finish the calculation)}
\end{align*}
\]