

Math 160 Sp'10 Lowman, Week 12 Wed

Note Title

4/8/2010

- Probability Review (cont.)

#10 practice final

Given:

$$P(A) = .5, P(\bar{B}) = .6, P(\bar{A} \cap \bar{B}) = .1$$

- Determine if A and B are independent.
- Determine if A and B are mutually Exclusive.

Note: tables, trees, Venn Diagrams are often useful for organizing.

- Trees are often useful when conditional probabilities are given
- Tables are often useful when probabilities of intersections are given.
- Venn Diagrams are essentially the same as tables.

First fill in the table then finish the problem $\bigcirc \Rightarrow$ Given values

	B	\overline{B}	
A	0	.5	\bigcirc .5
\overline{A}	.4	\bigcirc .1	.5
	.4	\bigcirc .6	\bigcirc 1

(a) Are A and B independent?

Test for independence:

$$\text{IF } P(A \cap B) = P(A) \cdot P(B)$$

Then A and B are independent.

Otherwise A and B are dependent.

Test:

$$P(A \cap B) = 0$$
$$P(A) \cdot P(B) = (.5)(.4) = .2$$

$P(A \cap B) \neq P(A) \cdot P(B)$ so A and B are not independent. They are dependent.

(b) are A and B mutually exclusive?

A and B are mutually exclusive if either $A \cap B = \emptyset$

or $P(A \cap B) = 0$.

From the table $P(A \cap B) = 0$ so
A and B are mutually exclusive.

Important observation:

If A and B are independent
then so are A, \bar{A}, B and \bar{B} .

So $P(A \cap B) = P(A) \cdot P(B)$

$$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$$

$$P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$$

If one pair is independent then all pairs are independent.

The same is not true for mutually exclusive events.

In this example $P(A \cap B) = 0$
so A and B are mutually
exclusive.

$P(A \cap \bar{B}) \neq 0$ so A and \bar{B} are
not mutually exclusive.

This makes sense because:
 A, B mutually exclusive means

that if event A did occur
then B cannot occur. If B
cannot occur then \bar{B} did
occur, i.e. A and \bar{B} are not
mutually exclusive when
 A and B are mutually exclusive

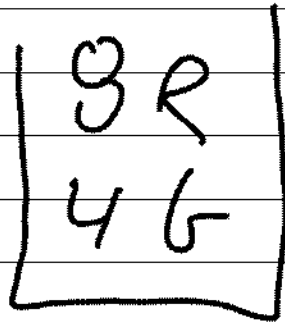
#11 practice final

*A box has 8 Red balls and 4 Green balls. Two are picked at random w/o replacement. Find the probability that the second ball is red given that a green ball is picked.

There are many ways to organize and work this problem. The probabilities of the 2nd ball picked are conditional so a tree will work. (There are simpler ways but a tree is easy and works).

Start by drawing pictures and tree.

$N=12$



$\frac{8}{12}$

R



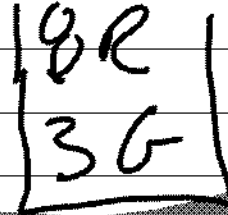
$\frac{7}{11}$

R

$\frac{4}{11}$

G

$N=11$



$\frac{4}{12}$

G

$\frac{8}{11}$

R

$\frac{3}{11}$

G

Note: all balls are equally probable so

the Classical Method can be used.

$$P(R_{2nd} | G_{1st}) = \frac{P(R_{2nd} \cap G_{1st})}{P(G_{1st})}$$

get probabilities from tree.

$$\begin{aligned} &= \frac{\frac{4}{12} \cdot \frac{8}{11}}{\frac{4}{12} \cdot \frac{8}{11} + \frac{4}{12} \cdot \frac{3}{11}} = \frac{\frac{4}{12} \cdot \frac{8}{11}}{\frac{4}{12} \left(\frac{8}{11} + \frac{3}{11} \right)} \\ &= \frac{8}{11} \end{aligned}$$