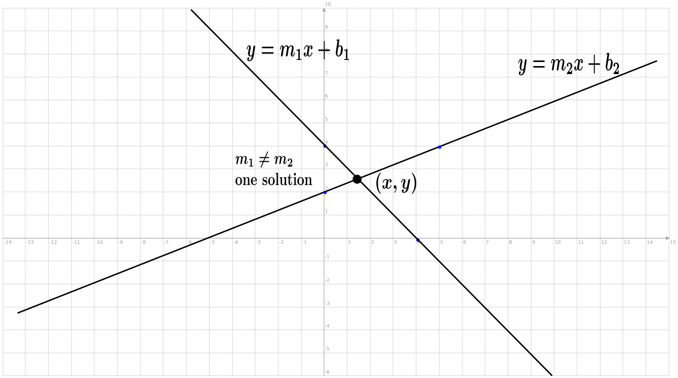
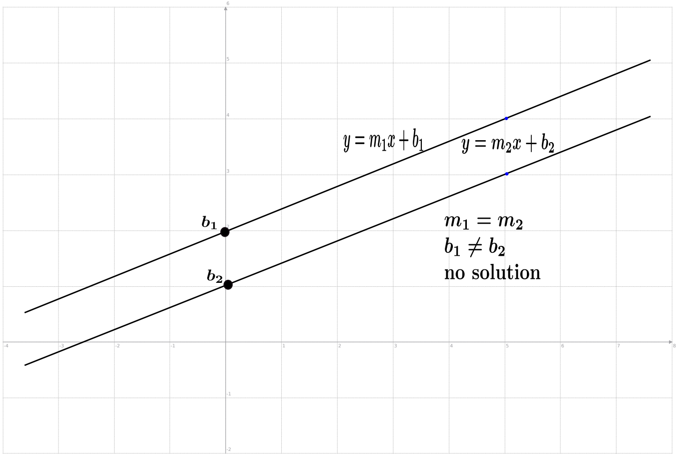
Gaussian Elimination - two lines

case 1

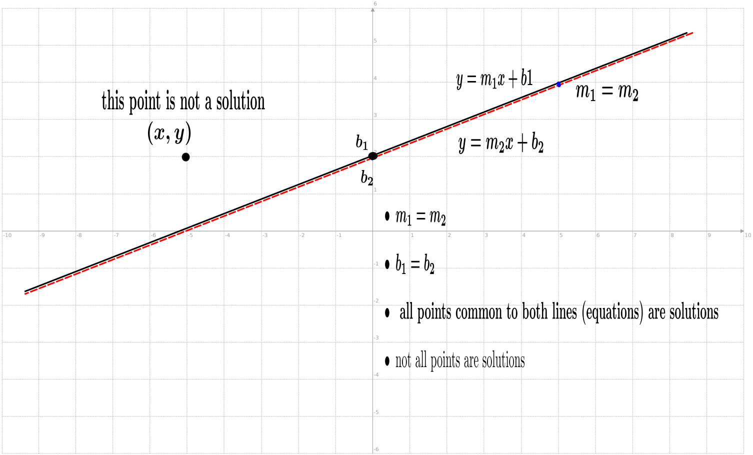


Gaussian Elimination - two lines

case 2



Gaussian Elimination - two lines
case 3



$$\mathbf{y} = \mathbf{m_1 x} + \mathbf{b_1} \quad (1)$$
$$\mathbf{y} = \mathbf{m_2 x} + \mathbf{b_2} \quad (2)$$

Three cases:

1. Case I: one solution
   - $\mathbf{m_1} \neq \mathbf{m_2}$, slopes different $\Rightarrow$ cross at one point.
2. Case II: no solution
   - $\mathbf{m_1} = \mathbf{m_2}$, slopes same $\Rightarrow$ parallel lines.
   - $\mathbf{b_1} \neq \mathbf{b_2}$ y-intercepts not same $\Rightarrow$ no common points.
3. Case III: infinite number of solutions. Must find all of them.
   - $\mathbf{m_1} = \mathbf{m_2}$, slopes same $\Rightarrow$ parallel lines.
   - $\mathbf{b_1} = \mathbf{b_2}$ y-intercepts are same $\Rightarrow$ all points common to both lines.
   - all points common to both lines are solutions.

$$2x + 3y = 8 \quad (3)$$
$$6x - 2y = 2 \quad (4)$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 2 & 3 & 8 \\ 6 & -2 & 2 \end{bmatrix} \qquad \begin{matrix} -- \\ R2- > \frac{1}{2}R2 \end{matrix}$$

$$\begin{bmatrix} 2 & 3 & 8 \\ 3 & -1 & 1 \end{bmatrix} \qquad \begin{matrix} -- \\ R_2 \longrightarrow R_2 + (-1) \cdot R_1 \end{matrix}$$

$$\begin{bmatrix} 2 & 3 & 8 \\ 1 & -4 & -7 \end{bmatrix} \qquad \begin{matrix} R_1 \leftrightarrow R_2 \\ -- \end{matrix}$$

$$\begin{bmatrix} 1 & -4 & -7 \\ 2 & 3 & 8 \end{bmatrix} \qquad \begin{matrix} -- \\ R_2 \longrightarrow R_2 + (-2) \cdot R_1 \end{matrix}$$

Gauss-Jordan Elimination

Case I: one solution cont.

$$\begin{bmatrix} 1 & -4 & -7 \\ 0 & 11 & 22 \end{bmatrix} \quad \begin{matrix} -- \\ R_2 \rightarrow \frac{1}{11}R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & -4 & -7 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_1 \rightarrow R_1 + (4) \cdot R_2 \\ -- \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{rref}$$

$$1x + 0y = 1 \tag{5}$$

$$0x - 1y = 2 \tag{6}$$

Gives the only solution $\boxed{(x,y) = (1,2)}$

Same Problem
different operations

$$2x + 3y = 8 \qquad (7)$$
$$6x - 2y = 2 \qquad (8)$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 2 & 3 & 8 \\ 6 & -2 & 2 \end{bmatrix} \qquad \begin{array}{l} R_1 \longrightarrow \frac{1}{2}R_1 \\ R_2 \longrightarrow \frac{1}{2}R_2 \end{array}$$

$$\begin{bmatrix} 1 & 3/2 & 4 \\ 3 & -1 & 1 \end{bmatrix} \qquad \begin{array}{l} -- \\ R_2 \longrightarrow R_2 + (-3) \cdot R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3/2 & 4 \\ 0 & -11/2 & -11 \end{bmatrix} \qquad \begin{array}{l} -- \\ R_2 \longrightarrow \frac{(-2)}{11}R_2 \end{array}$$

Same Problem
different operations (cont.)

$$\begin{bmatrix} 1 & 3/2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \qquad \begin{array}{l} R_1 \longrightarrow R_1 + (-3/2) \cdot R_2 \\ -- \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \qquad \begin{array}{l} x = 1 \\ y = 2 \end{array}$$

Any sequence of elementary row operations can be used to reach rref.

$$y = 2x + 2 \tag{9}$$
$$y = 2x + 1 \tag{10}$$

$$2x - y = -2 \tag{11}$$
$$-2x + y = 1 \tag{12}$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 2 & -1 & -2 \\ -2 & 1 & 1 \end{bmatrix} \quad \begin{matrix} -- \\ R_2 \longrightarrow R_2 + (1) \cdot R_1 \end{matrix}$$

$$\begin{bmatrix} 2 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{matrix} -- \\ \longrightarrow \text{no solution} \end{matrix}$$

Gauss-Jordan Elimination

Case III: infinite solutions

$$y = 2x + 1 \tag{13}$$
$$2y = 4x + 2 \tag{14}$$

$$2x - y = -1 \tag{15}$$
$$4x - 2y = -2 \tag{16}$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 2 & -1 & -1 \\ 4 & -2 & -2 \end{bmatrix} \qquad \begin{matrix} -- \\ R_2 \to R_2 + (-2) \cdot R_1 \end{matrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} -- \\ R_1 \to \frac{1}{2} R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{rref}$$

Equivalent system:

$$\mathbf{1x} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{y} = -\frac{\mathbf{1}}{\mathbf{2}} \tag{17}$$

$$\mathbf{0x} + \mathbf{0y} = \mathbf{0} \tag{18}$$

System with two equations in two unknowns reduced to one equation in two unknowns.
Gauss-Jordan $\longrightarrow$ solve for leading variables ($\mathbf{x}$) in terms of non-leading variables ($\mathbf{y}$).

$$\mathbf{x} = -\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{y} \tag{19}$$

$$\mathbf{y} = \mathbf{y}, \quad \mathbf{y} \text{ is any real number (since no restrictions on } \mathbf{y}) \tag{20}$$

Give a few solutions by picking a few y's and solving for x's giving points on both lines.