Gaussian Elimination: three equations, three unknowns

Use the Gauss-Jordan Elimination method to solve systems of linear equations.

1. Write corresponding augmented coefficient matrix
2. reduce to reduced row echelon form (rref), using three elementary row operations
3. from reduced matrix write the equivalent system of equations
4. solve for leading variables in terms of non-leading variables (if any)
5. set non-leading variables to any real number
6. write solution to system in matrix form. This is not part of G-J but is required for exam 1

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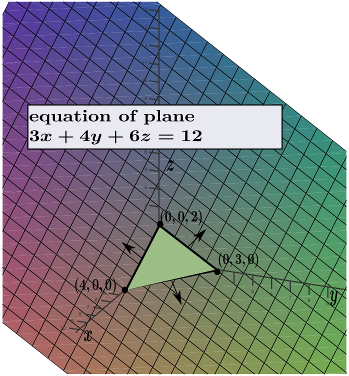
equation of plane and graph



Equations of form $\mathbf{ax + by + cz = d}$ graph as a plane in three dimensional space

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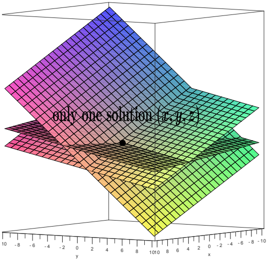
case I: one solution



$$
\begin{aligned}
x + 2y + z &= 5 & (1) \\
2x + y + 2z &= 7 & (2) \\
x + 2y + 4z &= 4 & (3)
\end{aligned}
$$

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case I: one solution

Use Matlab or free matlab clones.
Both Octave and FreeMat are similar to Matlab and are free downloads.

$$x + 2y + z = 5 \tag{4}$$
$$2x + y + 2z = 7 \tag{5}$$
$$x + 2y + 4z = 4 \tag{6}$$

Here Octave is used to reduce the system.

```
octave:22> #--------------system with one solution---------------------
octave:23> AB1 = [1 2 1 5;2 1 2 7;1 2 4 4]
AB1 =

       1       2       1       5
       2       1       2       7
       1       2       4       4

octave:25> rref(AB1)
ans =

       1       0       0    10/3
       0       1       0       1
       0       0       1    -1/3
```

$$x = \frac{10}{3}$$
$$y = 1$$
$$z = -\frac{1}{3}$$

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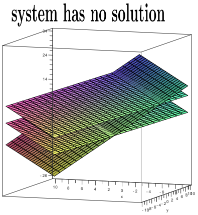
case II: no solution



$$x + y + z = 3 \quad (7)$$

$$2x + y + z = 4 \quad (8)$$

$$x + y + z = 10 \quad (9)$$

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case II: no solution

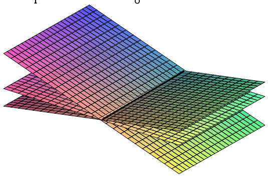
$$x + y + z = 3 \quad (10)$$
$$2x + y + z = 4 \quad (11)$$
$$x + y + z = 10 \quad (12)$$

```
octave:27> #--------------system with no solution---------------------
octave:28>AB2 = [1 1 1 3; 2 1 1 4; 1 1 1 10]
AB2 =

      1      1      1      3
      2      1      1      4
      1      1      1     10

octave:30>octave:31> rref(AB2)
ans =

      1      0      0      0
      0      1      1      0
      0      0      0      1
```

$$0 = 1 \Rightarrow \text{ no solution}$$

Gaussian Elimination: three equations, three unknowns

case III: infinite number of solutions



$$x + 2y + z = 5 \tag{13}$$
$$2x + y + 2z = 7 \tag{14}$$
$$x + y + z = 4 \tag{15}$$

Gaussian Elimination: three equations, three unknowns

case III: infinite number of solutions

$$x + 2y + z = 5 \quad (16)$$

$$2x + y + 2z = 7 \quad (17)$$

$$x + y + z = 4 \quad (18)$$

```
octave:32>#-------------system with infinite number of solutions--------------------
octave:33> AB3 = [1 2 1 5;2 1 2 7;1 1 1 4]
AB3 =

   1   2   1   5
   2   1   2   7
   1   1   1   4

octave:35> rref(AB3)
ans =

   1   0   1   3
   0   1   0   1
   0   0   0   0
```

$$x + z = 3$$

$$y = 1$$

x and y are leading variables or dependent variables

z is a non-leading or independent variable

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case III: infinite number of solutions

$$x + 2y + z = 5 \quad (19)$$
$$2x + y + 2z = 7 \quad (20)$$
$$x + y + z = 4 \quad (21)$$

Reduces to

$$x + 0y + z = 3 \quad (22)$$
$$0x + y + 0z = 1 \quad (23)$$

Solve for leading variables in terms of non-leading variables:

$$x = 3 - z \quad (24)$$
$$y = 1 \quad (25)$$
$$z = z, \quad \text{where} \quad z = \text{ any real number} \quad (26)$$

Here is the solution in matrix form:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{3} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix} + \mathbf{z} \begin{bmatrix} \mathbf{-1} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix}, \quad \text{z is any real number}$$