Math 160 Special Assignment #1 Lowman Spring 2010

- Due Thursday, Feb 11 in discussion time
- Calculators can be used to do matrix calculations only where specified. Otherwise, all calculations must be done by hand and you must show all work. When using elementary row operations, you must show your work using the same method that was used in lectures (see web notes).

1. Use the graphical method to solve the linear programming problem. You must graph the area of feasible solutions then list all corner points and z values in a table. Show all work.

$$\begin{array}{ll} \text{Maximize} & \boldsymbol{z = 6x + 5y} \\ \text{Subject to} & \boldsymbol{3x + 2y \le 120} \\ & \boldsymbol{4x + 6y \le 260} \\ & \boldsymbol{x, y \ge 0} \end{array}$$

2. Use the graphical method to solve the linear programming problem. A truck traveling from New York to Baltimore is to be loaded with two types of cargo. Each crate of cargo A is **8** cubic feet in volume, weighs **100** pounds, and earns **$10** for the driver. Each crate of cargo B is **4** cubic feet in volume, weighs **200** pounds, and earns **$12** for the driver. The truck can carry no more than **400** cubic feet of crates and no more than **10,000** pounds. Also, the number of crates of cargo A must be less than or equal to twice the number of crates of cargo B. Maximize drivers earnings subject to constraints.

3. Write the following matrix equation as an equivalent matrix equation of the form $\boldsymbol{A \cdot X = B}$

$$\boldsymbol{x} \cdot \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + \boldsymbol{y} \cdot \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \boldsymbol{z} \cdot \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

4. Write the following matrix equation as an equivalent matrix equation of the form $\boldsymbol{A \cdot X = B}$. Show that the dimensions are consistent.
$\boldsymbol{2x + 4y + z + 3w = 10}$
$\boldsymbol{x + 2y + 2z + 3w = 8}$
$\boldsymbol{x + 2y + z + 2w = 6}$

5. Use the Gauss-Jordan elimination method to solve the system and write your answer in matrix form:
$\boldsymbol{2x + 4y + z + 3w = 10}$
$\boldsymbol{x + 2y + 2z + 3w = 8}$
$\boldsymbol{x + 2y + z + 2w = 6}$

6. Use the Gauss-Jordan elimination method to solve the system and write your answer in matrix form:
$\boldsymbol{2x + 4y + z + 3w = 10}$
$\boldsymbol{x + 2y + 2z + 3w = 8}$
$\boldsymbol{x + 2y + z + 2w = 5}$

7. Use the Gauss-Jordan elimination method to solve the system and write your answer in matrix form:
$\boldsymbol{x + z = 3}$
$\boldsymbol{y + 2z = 5}$
$\boldsymbol{x + 2z = 0}$

8. Solve the following by first using the Gauss Jordan Elimination method to find the inverse of the coefficient matrix and then using $\boldsymbol{X = A^{-1} \cdot B}$ to solve $\boldsymbol{AX = B}$
$\boldsymbol{x + y + 0z = 2}$
$\boldsymbol{0x + y + 0z = 1}$
$\boldsymbol{x + 0y + z = 6}$

9. Given the matrices A and B find the element in the 3rd row and 4th column of matrix C=AB. Do not find all of the elements in matrix C. Show your work.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 0 \\ 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 0 & 1 & 2 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 & 9 & 0 \end{bmatrix} \tag{1}$$

10. (Calculator Problem) Given the matrices A, B and C solve the matrix equation $\boldsymbol{AX = 2BX + 3X + C}$ for the matrix $\boldsymbol{X}$ by doing: (a) use matrix algebra to find an expression for X (show work for each step) then (b) use your calculator to do the calculations.
$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} 90 \\ 100 \end{bmatrix}$, answer to (b): $\boldsymbol{X} = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$

11. (Calculator Problem) An economy consists of three sectors agriculture, energy, and manufacturing. For each \$1.00 worth of output, the agriculture sector requires \$.08 worth of input from the agriculture sector, \$.10 worth of input from the energy sector, and \$.20 worth of input from the manufacturing sector. For each \$1.00 worth of output, the energy sector requires \$.15 worth of input from the agriculture sector, \$.14 worth of input from the energy sector, and \$.10 worth of input from the manufacturing sector. For each \$1.00 worth of output, the manufacturing sector requires \$.25 worth of input from the agriculture sector, \$.12 worth of input from the energy sector, and \$.05 worth of input from the manufacturing sector.

    (a) Give the input-output matrix A for this economy.
    (b) Determine the matrix $\boldsymbol{(I - A)^{-1}}$. (Round entries to two decimal places.)
    (c) At what level of output should each sector produce to meet the demand for \$4 billion worth of agriculture, \$3 billion worth of energy, and \$2 billion worth of manufacturing?

12. Given set $\boldsymbol{A = \{a, b, c, d\}}$

    (a) List all subsets of A by following this procedure: **1st** list all subsets with no elements, **2nd** list all subsets with 1 element, **3rd** list all subsets with 2 elements, **4th** list all subsets with 3 elements, **5th** list all subsets with 4 elements.
    (b) count the number of subsets in each of the four steps then add to find the total number of subsets.

13. Given set $\boldsymbol{B = \{1, 2, 3, 4\}}$

    (a) List all subsets of B by following this procedure: starting with an empty set, build a subset by **1st** deciding if **1** goes in the subset (if yes put **1** in the new set), **2nd** decide if **2** goes in the subset (if yes put **2** in the new set), **3rd** decide if **3** goes in the subset (if yes put **3** in the new set), **4th** decide if **4** goes in the subset (if yes put **4** in the new set).

    (b) Count the number of subsets. If you follow this process in all possible ways, you should find **16** subsets.

14. Given the set $\boldsymbol{C = \{a, b, c, d, e, f, g\}}$

    (a) Use "*The Fundamental Principle of Counting*", also called the product rule, to determine how many subsets can be found from $\boldsymbol{C}$. Do not list the sets, just determine how many.

    (b) How many subsets of $\boldsymbol{C}$ have at least two elements? Hint: determine the total number of subsets then subtract the number of subsets that have one or less elements.