Solving System of Two equations
Graphical Method

$$y = x + 1 \tag{1}$$
$$y = -x + 5 \tag{2}$$

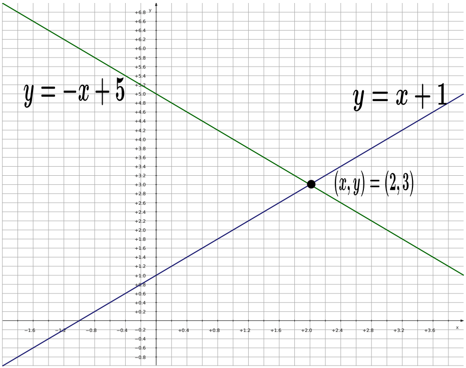
$$\mathbf{y = x + 1} \tag{1}$$
$$\mathbf{y = -x + 5} \tag{2}$$

are the same equations as:

$$\mathbf{x - y = -1} \tag{3}$$
$$\mathbf{x + y = 5} \tag{4}$$

Graphical Method

Solve by graphing equations and finding all common points.



$\boxed{\textbf{SubstitutionMethod}}$

$$\mathbf{x - y = -1} \tag{5}$$
$$\mathbf{x + y = 5} \tag{6}$$

Substitution Method

- solve for $\mathbf{x}$ in first equation

SubstitutionMethod

$$\mathbf{x - y = -1} \tag{5}$$
$$\mathbf{x + y = 5} \tag{6}$$

Substitution Method

- solve for **x** in first equation
- use to eliminate **x** in second equation

$\boxed{\textbf{SubstitutionMethod}}$

$$\mathbf{x} - \mathbf{y} = -\mathbf{1} \tag{5}$$
$$\mathbf{x} + \mathbf{y} = \mathbf{5} \tag{6}$$

Substitution Method

- solve for **x** in first equation
- use to eliminate **x** in second equation
- solve for remaining variable

SubstitutionMethod

$$\mathbf{x} - \mathbf{y} = -\mathbf{1} \tag{5}$$
$$\mathbf{x} + \mathbf{y} = \mathbf{5} \tag{6}$$

Substitution Method

- solve for **x** in first equation
- use to eliminate **x** in second equation
- solve for remaining variable
- use found variable in any equation to find other variable.

$$\mathbf{x - y = -1} \tag{7}$$
$$\mathbf{x + y = 5} \tag{8}$$

Substitution Method:

Solving System of Two equations
Substitution Method (2)

$$\mathbf{x - y = -1} \tag{7}$$

$$\mathbf{x + y = 5} \tag{8}$$

Substitution Method: From first equation

$$\mathbf{x = y - 1} \tag{9}$$

$$\mathbf{x - y = -1} \tag{7}$$
$$\mathbf{x + y = 5} \tag{8}$$

Substitution Method: From first equation

$$\mathbf{x = y - 1} \tag{9}$$

substitute for **x** in second equation gives:

$$\mathbf{(y - 1) + y = 5} \tag{10}$$
$$\mathbf{2y = 6} \tag{11}$$
$$\mathbf{y = 3} \tag{12}$$

$$\mathbf{x - y = -1} \tag{7}$$
$$\mathbf{x + y = 5} \tag{8}$$

Substitution Method: From first equation

$$\mathbf{x = y - 1} \tag{9}$$

substitute for $\mathbf{x}$ in second equation gives:

$$\mathbf{(y - 1) + y = 5} \tag{10}$$
$$\mathbf{2y = 6} \tag{11}$$
$$\mathbf{y = 3} \tag{12}$$

Now back substitute to find $\mathbf{x}$:

$$\mathbf{x = y - 1 = 3 - 1 = 2} \tag{13}$$

Giving $\mathbf{(x, y) = (2, 3)}$ as the only solution.

Elimination Method

$$\mathbf{x - y = -1} \tag{14}$$

$$\mathbf{x + y = \quad 5} \text{ Add to eliminate } \mathbf{y} \tag{15}$$

$$\mathbf{2x + 0 = 4} \tag{16}$$

$$\mathbf{x = 2} \tag{17}$$

Elimination Method

$$\mathbf{x - y = -1} \tag{14}$$

$$\mathbf{x + y = \quad 5} \text{ Add to eliminate } \mathbf{y} \tag{15}$$

$$\mathbf{2x + 0 = 4} \tag{16}$$

$$\mathbf{x = 2} \tag{17}$$

$$\textbf{Back Substitute} \text{ to get} \tag{18}$$

$$\mathbf{2 - y = -1} \tag{19}$$

$$\mathbf{y = 3} \tag{20}$$

$$\Rightarrow \mathbf{(x, y) = (2, 3)} \tag{21}$$

Gauss-Jordan Elimination

new example

$$\boxed{\textbf{Gauss} - \textbf{Jordan} \quad \textbf{Elimination}}$$

$$\mathbf{2x + 3y = 8} \tag{22}$$

$$\mathbf{6x - 2y = 2} \tag{23}$$

$$\boxed{\textbf{Gauss} - \textbf{Jordan} \quad \textbf{Elimination}}$$

$$\begin{aligned} 2x + 3y &= 8 && (22) \\ 6x - 2y &= 2 && (23) \end{aligned}$$

Use the **first elementary row operation: interchange two rows**

$$\begin{aligned} 2x + 3y &= 8 \qquad R_1 \leftrightarrow R_2 && (24) \\ 6x - 2y &= 2 && (25) \end{aligned}$$

$$\boxed{\textbf{Gauss} - \textbf{Jordan} \quad \textbf{Elimination}}$$

$$2x + 3y = 8 \tag{22}$$
$$6x - 2y = 2 \tag{23}$$

Use the **first elementary row operation: interchange two rows**

$$2x + 3y = 8 \qquad R_1 \leftrightarrow R_2 \tag{24}$$
$$6x - 2y = 2 \tag{25}$$

Gives **Equivalent System**:

$$6x - 2y = 2 \tag{26}$$
$$2x + 3y = 8 \tag{27}$$

Gauss-Jordan Elimination

$$6x - 2y = 2 \tag{28}$$

$$2x + 3y = 8 \tag{29}$$

Gauss-Jordan Elimination

$$6x - 2y = 2 \tag{28}$$
$$2x + 3y = 8 \tag{29}$$

Use the **second elementary row operation: Multiply a row by a number**

$$6x - 2y = 2 \qquad R_1 \rightarrow \frac{1}{2}R_1 \tag{30}$$
$$2x + 3y = 8 \tag{31}$$

Gauss-Jordan Elimination

$$6x - 2y = 2 \tag{28}$$
$$2x + 3y = 8 \tag{29}$$

Use the **second elementary row operation: Multiply a row by a number**

$$6x - 2y = 2 \qquad R_1 \rightarrow \frac{1}{2}R_1 \tag{30}$$
$$2x + 3y = 8 \tag{31}$$

Gives **Equivalent System**:

$$3x - y = 1 \tag{32}$$
$$2x + 3y = 8 \tag{33}$$

Gauss-Jordan Elimination

$$3x - y = 1 \tag{34}$$

$$2x + 3y = 8 \tag{35}$$

Gauss-Jordan Elimination

$$3x - y = 1 \tag{34}$$
$$2x + 3y = 8 \tag{35}$$

Use the **third elementary row operation: add to a row some multiple of another row**

$$3x - y = 1 \qquad R_1 \to R_1 + () \cdot R_2 \tag{36}$$
$$2x + 3y = 8 \tag{37}$$

Gauss-Jordan Elimination

$$3x - y = 1 \tag{34}$$
$$2x + 3y = 8 \tag{35}$$

Use the **third elementary row operation: add to a row some multiple of another row**

$$3x - y = 1 \qquad R_1 \to R_1 + () \cdot R_2 \tag{36}$$
$$2x + 3y = 8 \tag{37}$$

$$3x - y = 1 \qquad R_1 \to R_1 + (-1) \cdot R_2 \tag{38}$$
$$2x + 3y = 8 \tag{39}$$

Gauss-Jordan Elimination

$$3x - y = 1 \tag{34}$$
$$2x + 3y = 8 \tag{35}$$

Use the **third elementary row operation: add to a row some multiple of another row**

$$3x - y = 1 \qquad R_1 \rightarrow R_1 + () \cdot R_2 \tag{36}$$
$$2x + 3y = 8 \tag{37}$$

$$3x - y = 1 \qquad R_1 \rightarrow R_1 + (-1) \cdot R_2 \tag{38}$$
$$2x + 3y = 8 \tag{39}$$

Gives **Equivalent System**:

$$x - 4y = -7 \tag{40}$$
$$2x + 3y = 8 \tag{41}$$

Gauss-Jordan Elimination

$$\mathbf{x - 4y = -7} \tag{42}$$

$$\mathbf{2x + 3y = 8} \tag{43}$$

Gauss-Jordan Elimination

$$x - 4y = -7 \tag{42}$$
$$2x + 3y = 8 \tag{43}$$

Use the **third elementary row operation**

$$x - 4y = -7 \tag{44}$$
$$2x + 3y = 8 \qquad R_2 \rightarrow R_2 + (-2) \cdot R_1 \tag{45}$$

Gauss-Jordan Elimination

$$\mathbf{x - 4y = -7} \tag{42}$$
$$\mathbf{2x + 3y = 8} \tag{43}$$

Use the **third elementary row operation**

$$\mathbf{x - 4y = -7} \tag{44}$$
$$\mathbf{2x + 3y = 8} \qquad \mathbf{R_2 \longrightarrow R_2 + (-2) \cdot R_1} \tag{45}$$

Gives **Equivalent System**:

$$\mathbf{x - 4y = -7} \tag{46}$$
$$\mathbf{0x + 11y = 22} \tag{47}$$

Gauss-Jordan Elimination

$$\mathbf{x} - \mathbf{4y} = -\mathbf{7} \tag{48}$$

$$\mathbf{0x} + \mathbf{11y} = \mathbf{22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**

Gauss-Jordan Elimination

$$\mathbf{x} - \mathbf{4y} = -\mathbf{7} \tag{48}$$

$$\mathbf{0x} + \mathbf{11y} = \mathbf{22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**
At this point typically use one of two methods to continue:

1. **Gaussian Elimination**

Gauss-Jordan Elimination

$$\mathbf{x - 4y = -7} \tag{48}$$
$$\mathbf{0x + 11y = 22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**
At this point typically use one of two methods to continue:

1. **Gaussian Elimination**
   - use last row to solve for y

Gauss-Jordan Elimination

$$\mathbf{x} - \mathbf{4y} = -\mathbf{7} \tag{48}$$

$$\mathbf{0x} + \mathbf{11y} = \mathbf{22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**

At this point typically use one of two methods to continue:

1. **Gaussian Elimination**
   - use last row to solve for y
   - back-substitute to solve for x

Gauss-Jordan Elimination

$$\mathbf{x} - \mathbf{4y} = -\mathbf{7} \tag{48}$$
$$\mathbf{0x} + \mathbf{11y} = \mathbf{22} \tag{49}$$

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At this point typically use one of two methods to continue:

1. **Gaussian Elimination**
   - use last row to solve for y
   - back-substitute to solve for x
2. **Gauss-Jordan Elimination**

Gauss-Jordan Elimination

$$\mathbf{x} - \mathbf{4y} = -\mathbf{7} \tag{48}$$

$$\mathbf{0x} + \mathbf{11y} = \mathbf{22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**
At this point typically use one of two methods to continue:

1. **Gaussian Elimination**
   - use last row to solve for y
   - back-substitute to solve for x
2. **Gauss-Jordan Elimination**
   - continue to reduce to **Reduced Row Echelon Form (rref)**

Gauss-Jordan Elimination

$$\mathbf{x - 4y = -7} \tag{48}$$
$$\mathbf{0x + 11y = 22} \tag{49}$$

System is in **Triangular Form** or **Row-Echelon Form**
At this point typically use one of two methods to continue:

1. **Gaussian Elimination**
   - use last row to solve for y
   - back-substitute to solve for x
2. **Gauss-Jordan Elimination**
   - continue to reduce to **Reduced Row Echelon Form (rref)**
   - solve for leading variables in terms of non-leading variables.

Gauss-Jordan Elimination

Continue with Gauss-Jordan Elimination

$$x - 4y = -7 \qquad (50)$$

$$0x + 11y = 22 \qquad R_2 \rightarrow \frac{1}{11} R_2 \qquad (51)$$

$$x - 4y = -7 \qquad R_1 \rightarrow R_1 + (4) \cdot R_2 \qquad (52)$$

$$0x + y = 2 \qquad (53)$$

$$x - 0y = 1 \qquad (54)$$

$$0x + y = 2 \qquad (55)$$

Continue with Gauss-Jordan Elimination

$$x - 4y = -7 \tag{50}$$

$$0x + 11y = 22 \qquad R_2 \longrightarrow \frac{1}{11} R_2 \tag{51}$$

$$x - 4y = -7 \qquad R_1 \longrightarrow R_1 + (4) \cdot R_2 \tag{52}$$

$$0x + y = 2 \tag{53}$$

$$x - 0y = 1 \tag{54}$$

$$0x + y = 2 \tag{55}$$

Giving $(x, y) = (1, 2)$