Use the Gauss-Jordan Elimination method to solve systems of linear equations.

1. Write corresponding augmented coefficient matrix
2. reduce to reduced row echelon form (rref), using three elementary row operations
3. from reduced matrix write the equivalent system of equations
4. solve for leading variables in terms of non-leading variables (if any)
5. set non-leading variables to any real number
6. write solution to system in matrix form. This is not part of G-J but is required for exam 1

$$x + 2y + z = 3 \tag{1}$$
$$z = 1 \tag{2}$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \rightarrow R_1 + (-1) \cdot R_2 \\ -- \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{rref}$$

Gauss-Jordan Elimination

Case III write solutions

Equivalent system:

$$\mathbf{x} + 2\mathbf{y} = 2 \quad (3)$$
$$\mathbf{z} = 1 \quad (4)$$

System reduces to two equations in three unknowns. Leading variables are **x** and **z**. **y** is a non-leading variable.
Gauss-Jordan $\longrightarrow$ solve for leading variables (**x** and **z**) in terms of non-leading variable (**y**).

$$\mathbf{x} = 2 - 2\mathbf{y} \quad (5)$$
$$\mathbf{y} = \mathbf{y}, \qquad \mathbf{y} \text{ is any real number} \quad (6)$$
$$\mathbf{z} = 1 \quad (7)$$

Give a few solutions by picking a few y's and solving for points (x,y,z) that satisfy the original equations.

Here is the solution in matrix form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{y is any real number}$$

Matrices
basics

Here is a matrix with dimension or size **2x3**, two rows and three columns.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

In symbols:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$\begin{array}{lll} a_{11} = 1 & a_{12} = 2 & a_{13} = 3 \\ a_{21} = 4 & a_{22} = 5 & a_{23} = 6 \end{array}$$

$\mathbf{a_{ij}}$ is the element in row **i** and column **j**.

equality: Two matrices are equal if they have the same size and their corresponding components are equal.
addition: To add two matrices you add their corresponding components.
Example:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 8 & 7 \\ 6 & 5 \end{bmatrix} \tag{8}$$

$$\begin{aligned} \mathbf{C} &= \mathbf{A} + \mathbf{B} && (9) \\ &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 7 \\ 6 & 5 \end{bmatrix} && (10) \\ &= \begin{bmatrix} (1+8) & (2+7) \\ (3+6) & (4+5) \end{bmatrix} && (11) \\ &= \begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix} && (12) \end{aligned}$$

Matrix times number: To multiply a matrix by a number, multiply every element by the number. Example:

$$\mathbf{D} = 2 \cdot \mathbf{A} = 2 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \quad (13)$$

Use to factor a number out of a matrix.

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = 2 \cdot \frac{1}{2} \cdot \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = 2 \cdot \begin{bmatrix} \frac{1}{2} \cdot 2 & \frac{1}{2} \cdot 4 \\ \frac{1}{2} \cdot 6 & \frac{1}{2} \cdot 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad (14)$$

example:

$$\begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix} = 9 \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (15)$$