Math 165, Sp'12, Lowman Week 11, Friday

- Anti-Derivative, Indefinite Integral cont.

\[
\frac{dF(x)}{dx} = f(x) \quad \Rightarrow \quad \text{Same}
\]

\[
\int f(x) \, dx = F(x) + c
\]

for each derivative rule can find the corresponding integral rule.

This was covered in previous lecture.
\[ \frac{d}{dx} x^n = n \cdot x^{n-1} \quad \text{\{Simple Power Rule\}} \]
\[ x^n \, dx = \frac{x^{n+1}}{n+1} + c \]

Use chain rule to get general version.

\[ \frac{d}{dx} (u(x))^n = \frac{d}{dx} u^n \cdot \frac{du}{dx} \]
\[ = n \cdot u^n(x) \cdot u'(x), \quad \text{gives} \]
General Power Rule
\[
\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)
\]

\[
\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + c
\]

Note: In the special case where \( u(x) = x \), \( u'(x) = \frac{dx}{dx} = 1 \) and General Rules reduce to simple rules.
Observation: you can convert a simple integral rule to a general rule by replacing \( x \) with \( u \)
and \( dx \) with \( u \, du \).

Example

Simple Power Rule:

\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \\
\int u^n \, u \, du = \frac{u^{n+1}}{n+1} + c
\]
Summary of General Rules

\[
\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x) \quad \text{Power Rules}
\]

\[
\int u(x)^n \cdot u'(x) \, dx = \frac{u(x)^{n+1}}{n+1} + c
\]

\[
\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)} \quad \text{Log Rules}
\]

\[
\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + c
\]

\[
\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x) \quad \text{Exponential Rules}
\]

\[
\int u(x) \cdot e^{u(x)} \, dx = e^{u(x)} + c
\]
Example (that does not work!)

$$\int \frac{2x}{x+99} \, dx$$

Check if fits the Log Rule,

$$\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + c$$

Need the numerator to be the derivative of the denominator.

Check if \( (x+99) \) = \( 1 \)

This integral does not fit any of the above three rules. (skip for now)
Example:

\[ I = \int \frac{2x}{x^2 + 6} \, dx \]

\[ \text{Perfect fit.} \]

\[ \frac{d}{dx} (x^2 + 6) = 2x \]

\[ \text{Not Power Rule} \]

\[ \text{Not Exponential Rule} \]

\[ \text{Possible fit for Log Rule} \]

\[ \text{must check to be same.} \]

\[ \ln |u(x)| + C \]

\[ I = \ln |x^2 + 6| + c \]

Note: that \( x^2 + 6 \) is always positive so the absolute value can be dropped.
More Rules:

\[ \int [f(x) + g(x)] \, dx = \int f(x) \, dx + \int g(x) \, dx \]

\[ \frac{d}{dx} k \cdot u(x) = k \cdot \frac{d}{dx} u(x) \]

\[ \int k \cdot u(x) \, dx = k \int u(x) \, dx \]

\[ \int u(x) \, dx = \frac{1}{k} \int k \cdot u(x) \, dx \]

"multiply by one trick"
Example:

\[ I = \int x \cdot (2x^2 + 1)^3 \, dx \]

try Power Rule

\[ \int u^n \cdot u' \, dx = \frac{u^{n+1}}{n+1} + c \]

check \( d(2x^2 + 1) = 4x \) need a factor of \( 4 \) to fit the rule.

mutiply by \( \frac{1}{4} \)

\[ = \frac{1}{4} \int 4x \cdot (2x^2 + 1)^3 \, dx = \frac{1}{4} \left( \frac{2x+1}{4} \right)^4 + C = \frac{(2x+1)^4}{16} + C \]
Example:

Need a factor of \( y \).

\[
I = \int \frac{x}{2x^2 + 99} \, dx
\]

Check if fits Log Rule

\[
\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + c
\]

Check: \( \frac{d}{dx} (2x^2 + 99) = 4x \) always positive

\[
I = \frac{1}{4} \int \frac{4x}{2x^2 + 99} \, dx = \frac{1}{4} \ln |2x^2 + 99| + c
\]

\[
= \frac{1}{4} \ln (2x^2 + 99) + c
\]