Linear Functions

Graph of Line

Graph of line $y = mx + b$

$m = \text{slope}$

$b = \text{y-intercept}$
Linear Functions

x and y intercepts

Graph of line $y = mx + b$

y-int $(0, b)$

note $x = 0$ for y-intercept

x-int $(-\frac{b}{m}, 0)$

note $y = 0$ for x-intercept
Slope

\[ m = \frac{\text{rise}}{\text{run}} \]

\[ y = 2x + 1 \]

\[ \Delta y = y_2 - y_1 = 7 - 3 = 4 \]

\[ \Delta x = x_2 - x_1 = 3 - 1 = 2 \]

\[ m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2 \]
What is a slope?

definition

**Definition**

\[
\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Lines with large slopes change fast as \( x \) changes.

Lines with small slopes change slowly as \( x \) changes.

Lines with positive slopes increase as you go left to right.

Lines with negative slopes decrease as you go left to right.

Horizontal lines have slope = 0.

Vertical lines have slope undefined.
What is a slope?

definition

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Slopes of Lines
increase and decrease

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Slopes of Lines

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Slopes of Lines

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Slopes of Lines

Horizontal lines have slope \( = 0 \).

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Slopes of Lines

**horozontal lines**

- Horizontal lines have slope \( \text{slope} = 0 \).
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Slopes of Lines

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Slopes of Lines

vertical lines

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Slopes of Lines

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\[ (x_1, y_1) \quad (3, 1) \]

\[ (x_2, y_2) \quad \text{Vertical Line: } x = 3 \]

\[ \text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{4-1}{3-3} = \frac{3}{0} = \text{undefined} \]
Tangent Lines
from algebra to calculus

\[ y = f(x) = x^2 \]

Tangent Line at \((-1,1)\)
has slope \(m = -2\)

Tangent Line at \((1,1)\)
has slope \(m = +2\)

Tangent Line at \((0,0)\)
has slope \(m = 0\)
Equations of lines

Point-Slope Equation

Point-Slope Equation of Line, \( y - y_0 = m(x - x_0) \)
Equations of lines

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Slope-Intercept Equation for Line, \( y = mx + b \)
Equations of lines

Point-Slope Equation

\textbf{Slope-Intercept Equation for Line, } \quad y = mx + b

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Equations of lines
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1. Find equation of line with points \( P1 = (1, 1) \) and \( P2 = (3, 5) \)

\[ m = \frac{5-1}{3-1} = \frac{4}{2} = 2 \]

Use \( m \) and \( P1 \) with point-slope equation

\[ y - 1 = 2(x - 1) = 2x - 2 \]

Final expression in slope-intercept form
Find equation of line with points $P_1 = (1, 1)$ and $P_2 = (3, 5)$

- $m = \frac{5-1}{3-1} = \frac{4}{2} = 2$
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- final expression in slope-intercept form