Special Limits

definition of **e**

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1 + x)^{\frac{1}{x}}$						

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1+x)^{\frac{1}{x}}$	2.5937					

Special Limits

definition of **e**

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow 0$
$(1+x)^{\frac{1}{x}}$	2.5937	2.70481				

Special Limits

definition of **e**

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1 + x)^{\frac{1}{x}}$	2.5937	2.70481	2.71692			

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

$\mathbf{x}$	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1+x)^{\frac{1}{x}}$	2.5937	2.70481	2.71692	2.71814		

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow 0$
$(1+x)^{\frac{1}{x}}$	2.5937	2.70481	2.71692	2.71814	2.71826	

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1 + x)^{\frac{1}{x}}$	2.5937	2.70481	2.71692	2.71814	2.71826	$\longrightarrow \mathbf{e}$

Where $\mathbf{e} = \mathbf{2.7\ 1828\ 1828} \cdots$

Special Limits

definition of e

The number **e** is defined as a limit. Here is one definition:

$$\mathbf{e} = \lim_{x \longrightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

A good way to evaluate this limit is make a table of numbers.

x	.1	.01	0.001	0.0001	0.00001	$\longrightarrow \mathbf{0}$
$(1+x)^{\frac{1}{x}}$	2.5937	2.70481	2.71692	2.71814	2.71826	$\longrightarrow \mathbf{e}$

Where $\mathbf{e} = \mathbf{2.7\ 1828\ 1828} \cdots$

This limit will give the same result:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

Special Limits

alternate definition of **e**

Here is an equivalient definition for **e**:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^{x}$$

Special Limits

alternate definition of **e**

Here is an equivalient definition for **e**:

$$\mathbf{e} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
~~$(1+x)^{\frac{1}{x}}$~~ $\left(1+\frac{1}{x}\right)^x$					

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

x	100	1000	10000	1000000	$\to \infty$
$(1 + \frac{1}{x})^x$	2.70481				

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^{x}$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
$(1 + \frac{1}{x})^{x}$	2.70481	2.71692			

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
$(1 + \frac{1}{x})^{x}$	2.70481	2.71692	2.71815		

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x} \right)^{x}$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
$(1+x)^{\frac{1}{x}}$	2.70481	2.71692	2.71815	2.71827	

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^{x}$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
$(1 + x)^{\frac{1}{x}}$	2.70481	2.71692	2.71815	2.71827	$\longrightarrow \mathbf{e}$

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

x	100	1000	10000	1000000	$\to \infty$
$(1 + \frac{1}{x})^x$	2.70481	2.71692	2.71815	2.71827	$\to e$

Note that

$$\lim_{x \to 0} f(x)$$

is the same as

Special Limits

alternate definition of **e**

Here is an equivalent definition for **e**:

$$\mathbf{e} = \lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^{x}$$

x	100	1000	10000	1000000	$\longrightarrow \infty$
$(1 + \frac{1}{x})^{x}$	2.70481	2.71692	2.71815	2.71827	$\longrightarrow \mathbf{e}$

Note that

$$\lim_{x \longrightarrow 0} f(x)$$

is the same as

$$\lim_{x \longrightarrow \infty} f\left(\frac{1}{x}\right)$$

Special Limits

e the natural base

- the number **e** is the natural base in calculus. Many expressions in calculus are simpler in base **e** than in other bases like base $\mathbf{2}$ or base $\mathbf{10}$

Special Limits

e the natural base

- the number **e** is the natural base in calculus. Many expressions in calculus are simpler in base **e** than in other bases like base $\mathbf{2}$ or base $\mathbf{10}$
- $\mathbf{e = 2.718281828459045090 80 \cdots}$

Special Limits

e the natural base

- the number **e** is the natural base in calculus. Many expressions in calculus are simpler in base **e** than in other bases like base $\mathbf{2}$ or base $\mathbf{10}$
- $\mathbf{e = 2.718281828459045090 80 \cdots}$
- **e** is a number between $\mathbf{2}$ and $\mathbf{3}$. A little closer to $\mathbf{3}$.

Special Limits

e the natural base

- the number **e** is the natural base in calculus. Many expressions in calculus are simpler in base **e** than in other bases like base **2** or base **10**
- $\mathbf{e} = \mathbf{2.718281828459045090800}\cdots$
- **e** is a number between **2** and **3**. A little closer to **3**.
- **e** is easy to remember to **9** decimal places because **1828** repeats twice: $\mathbf{e} = \mathbf{2.718281828}$. For this reason, do not use **2.7** to extimate **e**.

Special Limits

e the natural base

- the number **e** is the natural base in calculus. Many expressions in calculus are simpler in base **e** than in other bases like base $\mathbf{2}$ or base $\mathbf{10}$
- $\mathbf{e = 2.718281828459045090\cdots}$
- **e** is a number between $\mathbf{2}$ and $\mathbf{3}$. A little closer to $\mathbf{3}$.
- **e** is easy to remember to $\mathbf{9}$ decimal places because $\mathbf{1828}$ repeats twice: $\mathbf{e = 2.718281828}$. For this reason, do not use $\mathbf{2.7}$ to extimate **e**.
- use the $\mathbf{e^x}$ button on your calculator to find **e**. Use $\mathbf{1}$ for **x**.

Special Limits

$\mathbf{e}$ the natural base

- the number $\mathbf{e}$ is the natural base in calculus. Many expressions in calculus are simpler in base $\mathbf{e}$ than in other bases like base $\mathbf{2}$ or base $\mathbf{10}$
- $\mathbf{e = 2.71828182845904509080 \cdots}$
- $\mathbf{e}$ is a number between $\mathbf{2}$ and $\mathbf{3}$. A little closer to $\mathbf{3}$.
- $\mathbf{e}$ is easy to remember to $\mathbf{9}$ decimal places because $\mathbf{1828}$ repeats twice: $\mathbf{e = 2.718281828}$. For this reason, do not use $\mathbf{2.7}$ to extimate $\mathbf{e}$.
- use the $\mathbf{e^x}$ button on your calculator to find $\mathbf{e}$. Use $\mathbf{1}$ for $\mathbf{x}$.
- example: $\mathbf{F = Pe^{rt}}$ is often used for calculating compound interest in business applications.

Special Limits

$f(x) = \frac{1}{x}$

Here are four useful limits:

- $$\lim_{x \to +\infty} \frac{1}{x} \to 0^+ = 0$$

Special Limits

$f(x) = \frac{1}{x}$

Here are four useful limits:

- $$\lim_{x \to +\infty} \frac{1}{x} \to 0^+ = 0$$
- $$\lim_{x \to -\infty} \frac{1}{x} \to 0^- = 0$$

Special Limits

$f(x) = \frac{1}{x}$

Here are four useful limits:

- $$\lim_{x \to +\infty} \frac{1}{x} \to 0^{+} = 0$$
- $$\lim_{x \to -\infty} \frac{1}{x} \to 0^{-} = 0$$
- $$\lim_{x \to 0^{+}} \frac{1}{x} \to +\infty$$

Special Limits

$f(x) = \frac{1}{x}$

Here are four useful limits:

- $$\lim_{x \longrightarrow +\infty} \frac{1}{x} \longrightarrow 0^+ = 0$$
- $$\lim_{x \longrightarrow -\infty} \frac{1}{x} \longrightarrow 0^- = 0$$
- $$\lim_{x \longrightarrow 0^+} \frac{1}{x} \longrightarrow +\infty$$
- $$\lim_{x \longrightarrow 0^-} \frac{1}{x} \longrightarrow -\infty$$

Special Limits

$\mathbf{f(x) = \frac{1}{x}}$ cont.

The general idea is this:

- $$\frac{1}{+\text{BIG}} \rightarrow +\text{small}$$

Special Limits

$\mathbf{f(x) = \frac{1}{x}}$ cont.

The general idea is this:

- $$\frac{1}{+\text{BIG}} \longrightarrow +\text{small}$$
- $$\frac{1}{-\text{BIG}} \longrightarrow -\text{small}$$

Special Limits

$f(x) = \frac{1}{x}$ cont.

The general idea is this:

- $$\frac{1}{+\text{BIG}} \longrightarrow +\text{small}$$
- $$\frac{1}{-\text{BIG}} \longrightarrow -\text{small}$$
- $$\frac{1}{+\text{small}} \longrightarrow +\text{BIG}$$

Special Limits

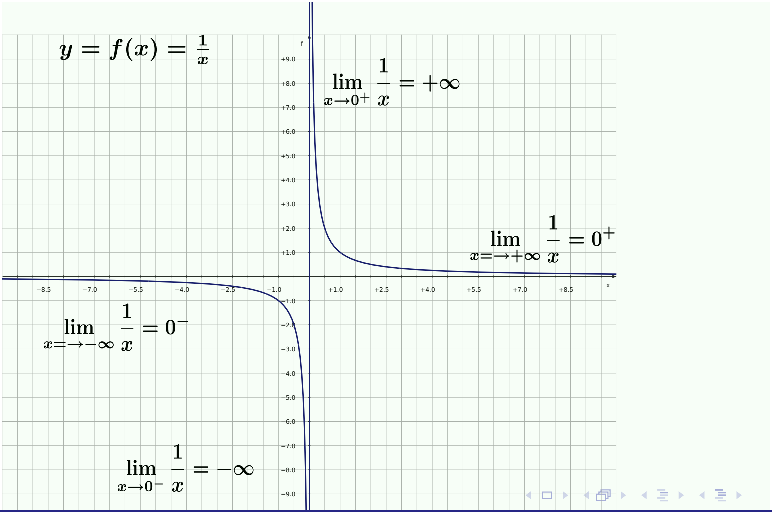
$f(x) = \frac{1}{x}$ cont.

The general idea is this:

- $$\frac{1}{+\text{BIG}} \longrightarrow +\text{small}$$
- $$\frac{1}{-\text{BIG}} \longrightarrow -\text{small}$$
- $$\frac{1}{+\text{small}} \longrightarrow +\text{BIG}$$
- $$\frac{1}{-\text{small}} \longrightarrow -\text{BIG}$$

Special Limits

$f(x) = \frac{1}{x}$



Special Limits

$\frac{1}{\text{BIG}} = \text{small}$, $\frac{1}{\text{small}} = \text{BIG}$

Look at $\frac{1}{x}$ using some numbers:

x	10	100	1000	10000	100000	$\to +\infty$
$f(x) = \frac{1}{x}$	.1	.01	.001	.0001	.00001	$\to 0^+$

x	-10	-100	-1000	-10000	-100000	$\to -\infty$
$f(x) = \frac{1}{x}$	-.1	-.01	-.001	-.0001	-.00001	$\to -0^+$

x	.1	.01	.001	.0001	.00001	$\to +0$
$f(x) = \frac{1}{x}$	10	100	1000	10000	100000	$\to +\infty$

x	-.1	-.01	-.001	-.0001	-.00001	$\to -0$
$f(x) = \frac{1}{x}$	-10	-100	-1000	-10000	-100000	$\to -\infty$

Special Limits

$x \to \pm\infty$ for Polynomials

- when taking limits of polynomials to $\pm\infty$ drop the lower degree terms and only keep the higest degree term of of the polymomial.

Special Limits

$x \to \pm\infty$ for Polynomials

- when taking limits of polynomials to $\pm\infty$ drop the lower degree terms and only keep the higest degree term of of the polymomial.
- this is an intermediate step in taking the limit. Use algebra to simplify the expression at this step then continue to work on finding the limit to infinity.

Special Limits

$\mathbf{x} \to \pm\infty$ for Polynomials

- when taking limits of polynomials to $\pm\infty$ drop the lower degree terms and only keep the higest degree term of of the polymomial.
- this is an intermediate step in taking the limit. Use algebra to simplify the expression at this step then continue to work on finding the limit to infinity.
- this works because for large values of $\mathbf{x}$ the highest power term of the polynomial is so much larger that all of the smaller degree terms that the smaller degree terms have no effect in the limit to infinity.

Special Limits

examples of $x \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x \longrightarrow +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3}$

Special Limits

examples of $x \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x \longrightarrow +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3}$

Special Limits

examples of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x \to +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} = \lim_{x \to +\infty} \frac{2x^3}{4x^3}$

Special Limits

examples of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x \to +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} = \lim_{x \to +\infty} \frac{2x^3}{4x^3} = \lim_{x \to +\infty} \frac{2}{4}$

Special Limits

examples of $x \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x\to+\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} = \lim_{x\to+\infty} \frac{2x^3}{4x^3} = \lim_{x\to+\infty} \frac{2}{4} = \lim_{x\to+\infty} \frac{1}{2}$

Special Limits

examples of $x \longrightarrow \pm\infty$ for Polynomials

- $\lim_{x \longrightarrow +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} = \lim_{x \longrightarrow +\infty} \frac{2x^3}{4x^3} = \lim_{x \longrightarrow +\infty} \frac{2}{4} = \lim_{x \longrightarrow +\infty} \frac{1}{2} = \frac{1}{2}$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\longrightarrow+\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3}$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\lim_{x \to +\infty} \frac{2x^3 + 99x + 1000}{1 + 2x^2 + 4x^3} = \lim_{x \to +\infty} \frac{2x^3}{4x^3}$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\lim_{x \to +\infty} \frac{2x^3 + 99x + 1000}{1 + 2x^2 + 4x^3} = \lim_{x \to +\infty} \frac{2x^3}{4x^3}$$

$$= \lim_{x \to +\infty} \frac{2}{4}$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \longrightarrow +\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} = \lim_{x \longrightarrow +\infty} \frac{2x^3}{4x^3}$$

$$= \lim_{x \longrightarrow +\infty} \frac{2}{4}$$

$$= \lim_{x \longrightarrow +\infty} \frac{1}{2}$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x\to+\infty} \frac{2x^3+99x+1000}{1+2x^2+4x^3} &= \lim_{x\to+\infty} \frac{2x^3}{4x^3} \\
&= \lim_{x\to+\infty} \frac{2}{4} \\
&= \lim_{x\to+\infty} \frac{1}{2} \\
&= \frac{1}{2}
\end{aligned}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \longrightarrow -\infty} \frac{2x^4 + 99x + 1000}{1 + 2x^2 + 4x^3}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \to -\infty} \frac{2x^4 + 99x + 1000}{1 + 2x^2 + 4x^3} = \lim_{x \to -\infty} \frac{2x^4}{4x^3}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\to-\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^3} = \lim_{x\to-\infty} \frac{2x^4}{4x^3}$$

$$= \lim_{x\to-\infty} \frac{2}{4}x$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \to -\infty} \frac{2x^4 + 99x + 1000}{1 + 2x^2 + 4x^3} = \lim_{x \to -\infty} \frac{2x^4}{4x^3}$$

$$= \lim_{x \to -\infty} \frac{2}{4} x$$

$$= \lim_{x \to -\infty} \frac{1}{2} x$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x \to -\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^3} &= \lim_{x \to -\infty} \frac{2x^4}{4x^3} \\
&= \lim_{x \to -\infty} \frac{2}{4}x \\
&= \lim_{x \to -\infty} \frac{1}{2}x \\
&= \frac{1}{2} \lim_{x \to -\infty} x
\end{aligned}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x\to-\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^3} &= \lim_{x\to-\infty} \frac{2x^4}{4x^3} \\
&= \lim_{x\to-\infty} \frac{2}{4}x \\
&= \lim_{x\to-\infty} \frac{1}{2}x \\
&= \frac{1}{2} \lim_{x\to-\infty} x \\
&= \frac{1}{2} \cdot -\infty
\end{aligned}$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\lim_{x\to-\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^3} = \lim_{x\to-\infty} \frac{2x^4}{4x^3}$$

$$= \lim_{x\to-\infty} \frac{2}{4}x$$

$$= \lim_{x\to-\infty} \frac{1}{2}x$$

$$= \frac{1}{2} \lim_{x\to-\infty} x$$

$$= \frac{1}{2} \cdot -\infty$$

$$= -\infty$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \longrightarrow +\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x \longrightarrow +\infty} \frac{2x^4 + 99x + 1000}{1 + 2x^2 + 4x^6} = \lim_{x \longrightarrow +\infty} \frac{2x^4}{4x^6}$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\rightarrow+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\rightarrow+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\rightarrow+\infty} \frac{2}{4x^2}$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\longrightarrow+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\longrightarrow+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\longrightarrow+\infty} \frac{2}{4x^2}$$

$$= \lim_{x\longrightarrow+\infty} \frac{1}{2x^2}$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\begin{aligned}\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} &= \lim_{x\to+\infty} \frac{2x^4}{4x^6} \\ &= \lim_{x\to+\infty} \frac{2}{4x^2} \\ &= \lim_{x\to+\infty} \frac{1}{2x^2} \\ &= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2}\end{aligned}$$

Special Limits

example of $x \rightarrow \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x\rightarrow+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} &= \lim_{x\rightarrow+\infty} \frac{2x^4}{4x^6} \\
&= \lim_{x\rightarrow+\infty} \frac{2}{4x^2} \\
&= \lim_{x\rightarrow+\infty} \frac{1}{2x^2} \\
&= \frac{1}{2} \lim_{x\rightarrow+\infty} \frac{1}{x^2} \\
&= \frac{1}{2} \cdot \frac{1}{+\infty}
\end{aligned}$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\to+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\to+\infty} \frac{2}{4x^2}$$

$$= \lim_{x\to+\infty} \frac{1}{2x^2}$$

$$= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2}$$

$$= \frac{1}{2} \cdot \frac{1}{+\infty}$$

$$= \frac{1}{2} \cdot 0^+ = 0^+ = 0$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\to+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\to+\infty} \frac{2}{4x^2}$$

$$= \lim_{x\to+\infty} \frac{1}{2x^2}$$

$$= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2}$$

$$= \frac{1}{2} \cdot \frac{1}{+\infty}$$

$$= \frac{1}{2} \cdot 0^+ = 0^+ = 0$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\to+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\to+\infty} \frac{2}{4x^2}$$

$$= \lim_{x\to+\infty} \frac{1}{2x^2}$$

$$= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2}$$

$$= \frac{1}{2} \cdot \frac{1}{+\infty}$$

$$= \frac{1}{2} \cdot 0^+ = 0^+ = 0$$

Special Limits

example of $\mathbf{x} \longrightarrow \pm\infty$ for Polynomials

$$
\begin{aligned}
\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} &= \lim_{x\to+\infty} \frac{2x^4}{4x^6} \\
&= \lim_{x\to+\infty} \frac{2}{4x^2} \\
&= \lim_{x\to+\infty} \frac{1}{2x^2} \\
&= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2} \\
&= \frac{1}{2} \cdot \frac{1}{+\infty} \\
&= \frac{1}{2} \cdot 0^+ = 0^+ = 0
\end{aligned}
$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} &= \lim_{x\to+\infty} \frac{2x^4}{4x^6} \\
&= \lim_{x\to+\infty} \frac{2}{4x^2} \\
&= \lim_{x\to+\infty} \frac{1}{2x^2} \\
&= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2} \\
&= \frac{1}{2} \cdot \frac{1}{+\infty} \\
&= \frac{1}{2} \cdot 0^+ = 0^+ = 0
\end{aligned}$$

Special Limits

example of $x \longrightarrow \pm\infty$ for Polynomials

$$\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} = \lim_{x\to+\infty} \frac{2x^4}{4x^6}$$

$$= \lim_{x\to+\infty} \frac{2}{4x^2}$$

$$= \lim_{x\to+\infty} \frac{1}{2x^2}$$

$$= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2}$$

$$= \frac{1}{2} \cdot \frac{1}{+\infty}$$

$$= \frac{1}{2} \cdot 0^+ = 0^+ = 0$$

Special Limits

example of $x \to \pm\infty$ for Polynomials

$$\begin{aligned}
\lim_{x\to+\infty} \frac{2x^4+99x+1000}{1+2x^2+4x^6} &= \lim_{x\to+\infty} \frac{2x^4}{4x^6} \\
&= \lim_{x\to+\infty} \frac{2}{4x^2} \\
&= \lim_{x\to+\infty} \frac{1}{2x^2} \\
&= \frac{1}{2} \lim_{x\to+\infty} \frac{1}{x^2} \\
&= \frac{1}{2} \cdot \frac{1}{+\infty} \\
&= \frac{1}{2} \cdot 0^+ = 0^+ = 0
\end{aligned}$$