Slope of Tangent

secant line
Slope of Tangent

secant line

\[ y = f(x) = x^2 \]

Need two points to find slope of tangent line but only know one \( (1, 1) \)
Slope of Tangent

secant line

1. Use secant to find slope of tangent.
2. Move the secant closer and closer to the tangent at (1,1).
3. Calculate the slope of each secant line.
4. Use the pattern of the slopes to determine the slope of the tangent.
Slope of Tangent

The slope of the tangent at point $(2, 4)$ is calculated as follows:

\[ \text{slope} = \frac{4-1}{2-1} = \frac{3}{1} = 3 \]
Slope of Tangent

secant line

\[ m = \frac{(1.96 - 1)}{(1.4 - 1)} = 2.4 \]
Slope of Tangent
secant line

\[ m = \frac{f(1.21) - f(1)}{1.21 - 1} = 2.1 \]

slope of secant

slope of tangent line is 2
As $x$ gets close to 1, the secant line gets close to the tangent line.

The function $y = f(x) = x^2$ is shown with a secant line and a tangent line at the point $(x, x^2)$.

The slope of the secant line is given by:

$$m(x) = \frac{\Delta y}{\Delta x} = \frac{x^2 - 1}{x - 1}$$

The point $(x, y) = (1, 1^2) = (1, 1)$ is marked on the graph.

The change in $x$ is $\Delta x = x_2 - x_1 = x - 1$.
Slope of Tangent

Right Hand Limit

<table>
<thead>
<tr>
<th>$x$</th>
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<th>1.5</th>
<th>1.1</th>
<th>1.01</th>
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The right hand limit (RHL) of $m(x) = \frac{x^2 - 1}{x - 1}$ as $x$ approaches 1 from the right is 2.
Slope of Tangent

Right Hand Limit

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► The right hand limit (RHL) of $m(x) = \frac{x^2 - 1}{x - 1}$ as $x$ approaches 1 from the right is 2.

► written

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$
Slope of Tangent

Right Hand Limit

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▶ The right hand limit (RHL) of $m(x) = \frac{x^2 - 1}{x - 1}$ as $x$ approaches $1$ from the right is $2$.

▶ written

$$\lim_{{x \rightarrow 1^+}} \frac{x^2 - 1}{x - 1} = 2$$

▶ It seems that the slope of the line tangent to $f(x) = x^2$ at $x = 1$ is $2$. 
**Slope of Tangent**

**Right Hand Limit**

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- The right hand limit (RHL) of $m(x) = \frac{x^2-1}{x-1}$ as $x$ approaches 1 from the right is 2.

- Written
  
  $$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$

- It seems that the slope of the line tangent to $f(x) = x^2$ at $x = 1$ is 2.

- It is often necessary to check if get the same result when $x$ approaches 1 from the left.
Slope of Tangent

secant line

- Repeat process from other side of point \((1,1)\)
- Find slope of secant lines thru points left of \((1,1)\) and \((1,1)\)
- Must get same tangent slope from left as did from right
The left hand limit (LHL) of $m(x) = \frac{x^2-1}{x-1}$ as $x$ approaches 1 from the left is 2.
The left hand limit (LHL) of $m(x) = \frac{x^2 - 1}{x - 1}$ as $x$ approaches 1 from the left is 2.

Written

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$
Slope of Tangent

Left Hand Limit

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<tr>
<th>$x$</th>
<th>0.9</th>
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- The left hand limit (LHL) of $m(x) = \frac{x^2 - 1}{x - 1}$ as $x$ approaches 1 from the left is 2.
- Written

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

- Again it seems that the slope of the line tangent to $f(x) = x^2$ at $x = 1$ is 2
Slope of Tangent

Existance of Limits

▶ The right hand limit exists and is 2

\[ \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2 \]
Slope of Tangent

Existence of Limits

- The right hand limit exists and is 2

\[
\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2
\]

- The left hand limit exists and is 2

\[
\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2
\]
Slope of Tangent

Existence of Limits

- The right hand limit exists and is 2
  \[ \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2 \]

- The left hand limit exists and is 2
  \[ \lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2 \]

- The LHL and RHL exist and are both equal to 2.
Slope of Tangent

Existance of Limits

- The right hand limit exists and is 2
  \[ \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2 \]

- The left hand limit exists and is 2
  \[ \lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2 \]

- The LHL and RHL exist and are both equal to 2.
- Therefore the two sided limit exists and is 2.
Slope of Tangent

Existence of Limits

- The right hand limit exists and is $2$
  \[ \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2 \]

- The left hand limit exists and is $2$
  \[ \lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2 \]

- The LHL and RHL exist and are both equal to $2$.
- Therefore the two sided limit exists and is $2$
- Written
  \[ \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2 \]