$f'(x) \Rightarrow f(x)$increasing/decreasing

Math165: Business Calculus

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Spring 2010, Week5 Lec1

- $f(x)$ is increasing at points where $\mathbf{f' > 0}$
- $f(x)$ is decreasing at points where $\mathbf{f' < 0}$
- **Critical Numbers** (CN, x_c) occur where $f'(x) = 0$ or $f'(x)$ **is undefined**.
- **Critical Numbers** are values of $x = x_c$ where $f(x)$ can change from *increasing to decreasing* or *decreasing to increasing*.
- If $f(x)$ is defined at x_c then the point on the graph $(x_c, f(x_c))$ is a **Critical Point (CP)**.
- In some cases, there is no point on the graph at a critical number x_c

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f' : increasing/decreasing
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One typical use for $\mathbf{f'}$
Where is $\mathbf{f(x)}$ inc/dec?

Use $\mathbf{f'(x)}$ to determine intervals where $\mathbf{f(x)}$ is increasing and where $\mathbf{f(x)}$ is decreasing.

1st find all critical numbers to determine **boundaries** on the graph where $\mathbf{f(x)}$ can change from increasing to decreasing etc.

- These boundaries, x_c, occur where $f'(x) = 0$ or $f'(x)$ is undefined.
- these boundaries are the only places where $f(x)$ can change from inc to dec or dec to inc

2nd determine the sign of $\mathbf{f'(x)}$ at one test value of $\mathbf{x}$ between each boundary

- if $f'(x) = (+)$ at this test value then it is increasing here and at all x in the same interval. It can only change at the boundaries given by the critical numbers.
- if $f'(x) = (-)$ at this test value then it is decreasing here and at all x in the same interval. It can only change at the boundaries given by the critical numbers.
- the function does not always change what it is doing across a critical number.

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Typical Exam problem:

For some $\mathbf{f(x)}$, $\mathbf{f'(x) = \frac{(2x-x^2)}{(x-3)}}$ use $\mathbf{f'}$ to determine where the graph of $\mathbf{f(x)}$ is increasing and where it is decreasing.

- Find all critical numbers where $\mathbf{f' = 0}$
- Solve $\mathbf{f'(x) = \frac{(2x-x^2)}{(x-3)} = 0}$.
- $\mathbf{f'(x)}$ is a rational function. It can be zero only for values of $\mathbf{x}$ where the numerator is zero and the denominator is NOT zero.
- $\Rightarrow$ solve:
  $\mathbf{2x - x^2 = 0}$
  $\mathbf{x(2 - x) = 0}$
  $\mathbf{x_c = 0, 2}$ from setting the slope = zero.

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  $(x-3) = 0$ giving $x_c = 3$
- Summary: there are three **CNs**, $x_c = 0, 2, 3$

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- Now test the sign of f' at one value in each interval.
- $f'(-1) = \frac{(-1)(2-(-1))}{((-1)-3)} = \frac{(-)(+)}{(-)} = (+) \Rightarrow f(x)$ is increasing at $x = -1$
- so $f(x)$ is increasing for all x in the interval $(-\infty, 0)$
- $f'(1) = \frac{(1)(2-(1))}{((1)-3)} = \frac{(+)(+)}{(-)} = (-) \Rightarrow f(x)$ is decreasing at $x = 1$
- so $f(x)$ is decreasing for all x in the interval $(0, 2)$
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- $\mathbf{f'(2.5) = \frac{(2.5)(2-(2.5))}{((2.5)-3)} = \frac{(+)(-)}{(-)} = (+) \Rightarrow f(x)}$ is increasing at $\mathbf{x = 2.5}$
- so $\mathbf{f(x)}$ is increasing for all $\mathbf{x}$ in the interval $\mathbf{(2, 3)}$
- $\mathbf{f'(4) = \frac{(4)(2-(4))}{((4)-3)} = \frac{(+)(-)}{(+)} = (-) \Rightarrow f(x)}$ is decreasing at $\mathbf{x = 4}$
- so $\mathbf{f(x)}$ is decreasing for all $\mathbf{x}$ in the interval $\mathbf{(3, +\infty)}$

$f'(x) = \frac{(2x - x^2)}{(x-3)} = \frac{x(2-x)}{(x-3)}$. There are three **CNs** $x_c = 0, 2, 3$

- Now test the sign of f' at one value in each interval.
- $f'(-1) = \frac{(-1)(2-(-1))}{((-1)-3)} = \frac{(-)(+)}{(-)} = (+) \Rightarrow f(x)$ is increasing at $x = -1$
- so $f(x)$ is increasing for all x in the interval $(-\infty, 0)$
- $f'(1) = \frac{(1)(2-(1))}{((1)-3)} = \frac{(+)(+)}{(-)} = (-) \Rightarrow f(x)$ is decreasing at $x = 1$
- so $f(x)$ is decreasing for all x in the interval $(0, 2)$
- $f'(2.5) = \frac{(2.5)(2-(2.5))}{((2.5)-3)} = \frac{(+)(-)}{(-)} = (+) \Rightarrow f(x)$ is increasing at $x = 2.5$
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One typical use for $\mathbf{f'}$
example continued

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example continued

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One typical use for $\mathbf{f'}$

example continued

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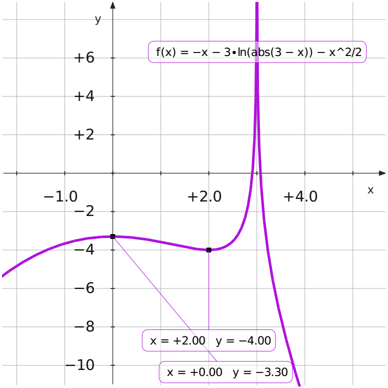

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