

6 $f(x) = (2x+1)^x$
 Logarithmic Differentiation
 $\ln f(x) = \ln[(2x+1)^x]$

$$\ln f(x) = x \cdot \ln(2x+1)$$

$$\frac{d}{dx} \ln f(x) = \frac{d}{dx} [x \cdot \ln(2x+1)]$$

$$\frac{f'(x)}{f(x)} = \cancel{\frac{d}{dx}} \cdot \ln(2x+1) + x \cdot \frac{d}{dx} \ln(2x+1)$$

$$\frac{f'(x)}{f(x)} = \ln(2x+1) + x \cdot \frac{\frac{d}{dx}(2x+1)^2}{(2x+1)}$$

$$\frac{f'(x)}{f(x)} = \ln(2x+1) + \frac{2x}{2x+1}$$

multiply both sides by $f(x)$ to get $f'(x)$

$$f'(x) = f(x) \left[\ln(2x+1) + \frac{2x}{2x+1} \right]$$

$$f'(x) = (2x+1)^x \left[\ln(2x+1) + \frac{2x}{2x+1} \right]$$

Now evaluate at $x=1$

$$f'(1) = (2 \cdot 1 + 1)^1 \cdot \left[\ln(2 \cdot 1 + 1) + \frac{2 \cdot 1}{2 \cdot 1 + 1} \right]$$

$$= 3 \cdot \left[\ln 3 + \frac{2}{3} \right]$$

$$f'(1) = \underbrace{3 \ln 3}_{\text{exact}} + 2 \underset{(3)}{\approx} \boxed{5.2958} \text{ to 4 decimal places.}$$