MCS 261  Homework 1  Lowman  Summer 2004

1. Give the truth tables for:  
   \( a \lor b \)  \( a \land b \)  \( a \rightarrow b \)  \( a \equiv b \)  \( a \rightarrow b \)  \( a \leftrightarrow b \)  
   Assuming the notation in your text explain the difference (a) between \( \rightarrow \) and \( \Rightarrow \) and (b) between \( \leftrightarrow \) and \( \Leftrightarrow \).

2. Starting with the truth table for \( a \rightarrow b \)  
   (a) explain what it means to prove \( a \rightarrow b \)  
   (b) what must you show to prove \( a \rightarrow b \)

3. Show that \( a \rightarrow b \equiv \neg a \lor b \) is a tautology.

4. Explain the following methods of proof:  
   (a) proof by deduction, (b) direct proof, (c) proof by contrapositive,  
   (d) proof by contradiction, (e) proof by cases and (f) disprove something by using a counter example.

5. Give an example for each proof method.

6. Prove that \( (a \rightarrow b) \equiv (\neg b \rightarrow \neg a) \) by (a) using a truth table and  
   (b) by using the laws of logic. Hint, start by finding an expression for \( \neg (a \rightarrow b) \)

7. Use a truth table to prove that \( (a \rightarrow b) \equiv (a \land \neg b) \) is a contradiction.

8. Give the truth table for \( a \rightarrow b \), its converse, inverse and contrapositive. 
   What do you conclude from the truth table?

9. Prove that the sum of two prime numbers larger than 2 is not a prime number.

10. Prove that if \( n \) is even then \( n^2 \) is even.

11. Prove that if \( n^2 \) is even then \( n \) is even.

12. Prove that "\( n \) is even" is necessary and sufficient for "\( n^2 \) is even".

13. Prove that if \( x \) is a positive even integer then either \( x \leq 2 \) or \( x \) is composite.

14. Given the predicate \( P(x) \), is it true that \( (\exists x, \neg P(x)) \equiv (\neg [\forall x, P(x)]) \) ?

15. Prove that the following proposition is true.  
   Let \( a, b, c \in \mathbb{Z} \) with \( a \neq 0 \). If \( a \mid b \) and \( a \mid c \) then \( a \mid (b + c) \)

16. Prove that \( \sqrt{2} \) is irrational.

17. Prove the proposition: If \( n \) is a positive composite number, then \( n \) has at  
   least one prime factor \( p \) with \( 1 < p \leq \sqrt{n} \)

18. Let \( T \) be the set of all nonnegative powers of 2 and \( F \) be the set of all 
   nonnegative powers of 4. Prove that \( F \subset T \). (Show that \( F \) is a proper 
   subset of \( T \))
19. Two sets $A$ and $B$ are equal if they have the same elements. One method to prove that $A = B$ is to show that $(A \subseteq B) \land (B \subseteq A)$.

Prove that $(A \cup B = A \cap B$)

You must show that the set on the left of the equal sign has the same elements as the set on the right.

(a) prove equality by using a venn diagram.

(b) prove equality by starting with $x \in A \cup B$ and showing that the left is a subset of the right. Then show that the right is a subset of the left.

20. Given $x$ is a real number, prove that $x^2 \leq 1 \iff -1 \leq x \leq 1$. 