Math 504 Set Theory I
Problem Set 1

Due Wednesday January 23

1) Prove that $\langle a, b \rangle = \langle c, d \rangle$ if and only if $a = c$ and $b = d$.

2) Prove that if A and B are sets, then so is their symmetric difference $A \triangle B$.

3) Prove that if A is a non-empty set, then

$$\bigcap_{B \in A} B = \{x : \forall B \in A \; x \in B\}$$

is a set.

What happens if $A = \emptyset$.

4) Prove that

A0)–A3 $\nvdash \exists x \exists u \exists v \exists w \; (u \in x \wedge v \in x \wedge w \in x \wedge u \neq v \wedge v \neq w \wedge u \neq w)$

i.e., A0)–A3) are not enough to prove that there is a set with at least three elements. [Hint: Let

$T_0 = \{\emptyset\}$

$T_{n+1} = \{\{x, y\} : x, y \in T_n\} \cup T_n$ and

$T = \bigcup_{n=0}^{\infty} T_n.$

Consider $\mathcal{M} = (T, \in)$.]